A NEW VISION FOR MATHEMATICS EDUCATION IN OHIO

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PREFACE

Ohio has a problem. Numerous research reports indicate that the great majority of Ohio students—like their peers all across America—are failing to become proficient in mathematics. Because the importance of mathematical understanding is increasing as the world becomes increasingly technological, the failure of Ohio mathematics education not only is taking a severe toll on students as they move into their adult lives, it threatens the economic viability of Ohio and America in a globally competitive world.

Almost everyone agrees that Ohio must do something about the mathematics education problem. However, there are two major reasons why past attempts at a solution have failed. First, Ohio is not guided by a clear statewide vision for how the mathematics education system should work. Second, policy and practice in Ohio mathematics education are not properly guided by scientific research on mathematics learning and teaching.

The goal of the mathematics education vision document is to create a scientifically sound description of how Ohio mathematics education can become genuinely effective for all students. This vision can be achieved only if it is fully informed by and based on a basic understanding of a scientific description of the problems of mathematics learning and teaching. Myths and tradition must be replaced by scientifically sound theories and findings. Scientific understanding of mathematics learning and teaching—which forms the core of all genuine solutions—must become the foundation for Ohio mathematics education.

As the vision document discusses issues in Ohio mathematics education, it highlights the scientific theories and research that form the foundation for understanding the problems and solutions. Although the vision document discusses policy—especially where current policies conflict with scientific research—it is not a "policy document." Instead, it discusses in detail the fundamentals on which sound policy must be based.

SECTION 1: THE NEED FOR A VISION

Educational Challenges in the 21st Century

Understanding As the Key to Dealing with the Rapidly Changing World

The world is changing at an ever increasing pace. The social, technological, and work environments that current students will encounter throughout their lifetimes will be significantly different from those of the present or past—offering new opportunities and posing new challenges.

To productively deal with tomorrow's opportunities and challenges, today's students must develop numerous intellectual skills that differ substantially from those needed in the past. Today's students must become fluent and adaptable problem solvers who can develop solutions to ever-changing sets of problems. They must become lifelong learners who can independently acquire new knowledge, skills, and reasoning. The development of these intellectual fluencies is especially important in mathematics, where advancing technology has caused many traditional skills to become obsolete and made other previously neglected skills basic.

At the core of the new basic skills in mathematics is understanding (Hiebert et. al., 1997). Understanding is critical because only ideas learned with genuine understanding can support flexible problem solving, can be adapted to new situations, and can serve as the foundation for new learning. Learning ideas with deep understanding is the intellectual foundation on which education for a fast-changing and unpredictable world must be built.

The Future of America Is at Stake

According to the United States Commission on National Security, (a) education is the foundation of America’s future, and (b) that future is in jeopardy (2001). The commission argues that, "the inadequacies of our systems of research and education pose a greater threat to U.S. national security over the next quarter century than any potential conventional war that we might imagine" (USCNS, 2001, p. ix). "Our systems of basic scientific research and education are in serious crisis….In the next quarter century, we will likely see ourselves surpassed, and in relative decline, unless we make a conscious national commitment to maintain our edge" (2001, p. ix). "The capacity of America’s educational system to create a 21st century workforce second to none in the world is a national security issue of the first order. As things stand, this country is forfeiting that capacity" (2001, p. 38). "There will not be enough qualified American citizens to perform the new jobs being created today—including technical jobs crucial to the maintenance of national security" (2001, p. 39). "Given the exigencies of advanced 21st century economies, it is not good enough that we produce a sufficient elite corps of science, math, and engineering professionals. We must raise levels of math, science, and technology literacy throughout our society" (2001, pp. 45-6).

Similarly, according to the Organization for Economic Cooperation and Development, evidence shows that business productivity is positively related to investment in education and training. To compete internationally, adapt to new technologies, and attain higher levels of efficiency and productivity, business and industry require highly skilled employees (OECD, 2000). To remain competitive in the new knowledge-based global economy, companies are increasingly utilizing management practices such as "multi-skilling," teamwork, reduced hierarchical levels, and delegation of responsibility to individuals and teams—all of which leads firms to demand more flexibility and higher levels of skills from the workforce (OECD, 2000). High level skills "are becoming increasingly important in the knowledge economy, both for individuals and at the macro level. Countries with higher levels of skills will adjust more effectively to challenges opened by globalisation because their firms will be more flexible and better able to absorb and adapt new technologies and to work with new equipment. The skill level and quality of the workforce will increasingly provide the cutting edge in competing in the global economy" (OECD, 2000, p. 11). Unfortunately, a recent OECD analysis concluded that the US has lost its international lead in educating workers for the knowledge-based economy. "Thirty years ago, the United States was the undisputed leader in educating its population….The United States not only lost the lead for participation in education, but the US has been overcome by countries doing a better job in regards to quality" (Mollison, 2001).

America's and Ohio's Economic Prosperity Is Dependent on ALL Students Properly Learning Mathematics

"Never before in history has the link between economic well-being and education been stronger" (Hershberg, 2000, p. 32).

Because success in the modern economy requires an appropriate labor force, no factor is currently more important to the economic well-being of nations and states than the capacity to develop their human resources (Hershberg, 2000). In fact, if America is to remain a "middle-class society" in the high-tech and global economy, schools "must graduate all their students—not merely the top fifth—with new and far higher skills than were necessary in the past" (Hershberg, 2000, p. 32). Furthermore, as former U.S. Secretary of Education, Richard Riley, stated, "Today's students must master high-level mathematical concepts and complex approaches to solving problems to be prepared for college, careers of the 21st century, and the demands of everyday life" (CBMS, 2000).

The new need for universal, high level mathematics education is a stark break from the past. For most of the 20th century, it was acceptable for schools to educate only the top fifth of students reasonably well because the poor performance of the other 80 percent did not matter much; when these students left school they entered a manufacturing economy with an abundant number of decent-paying unskilled jobs (Hershberg, 2000). Because the majority of students found jobs in a manufacturing economy that required little innovative thinking, the traditional instructional focus in America's K-12 schools, which is rooted in the view of learning as memorization, was sufficient.

Now, however, new technologies and industries favor better-educated workers who are adept at reasoning, problem solving, and learning; mere memorization is insufficient (Bransford, Brown, & Cocking, 1999; Hershberg, 2000). As a consequence, school mathematics programs must focus on developing students' mathematical reasoning and problem-solving skills, in addition to helping students build the intellectual wherewithal for further learning. In mathematics, students must learn to ask questions, make and test predictions, use evidence and logic to draw conclusions, and independently make sense of new information. And the basis for all these critical activities is, as suggested earlier, deep conceptual and principled understanding of mathematics.

> UC Ohio Survey Highlight. In 2000, a survey was completed in which 1,527 adults randomly selected to be representative of the Ohio population were asked questions about mathematics and science education in Ohio (UC 2000). A large majority of participants agreed that mathematics education has practical value: 92% agreed that mathematics had value in the workplace and the economy; and 94% agreed that improvements in mathematics education would improve children's job opportunities.

> UC Ohio Survey Highlight. Sixty-eight percent of respondents believed that basic mathematics skills have changed over the past 30 years. Importantly, 94% agreed that mathematics should help students make sense of the world around them. (Note that 29% of respondents believed that mathematics basics have not changed; so a large number of Ohioans need to be educated about the need for change.)

Problems with the Status Quo

Unfortunately, instead of keeping pace with the needs of a modern technological society, American mathematics education, including that in Ohio, is still clinging to outdated and inadequate curriculum goals and instructional methods (Battista, in press; van der Ploeg, 2001; Resnick, 1995). This has serious consequences both for individuals and the nation.

Scientific Research Indicates Poor U. S. Student Mathematics Achievement

Numerous studies have shown that U. S. students' mathematics learning is deficient, especially in the area of problem solving and reasoning (Beaton et al., 1996; Mullis, Dossey, Owen, & Phillips 1993; NCES, 1996; Hiebert, 1999). Indeed, results published by the National Assessment of Educational Progress indicate that only about 13-18% of grade 12 students are proficient in mathematics (Mullis, Dossey, Owen, & Phillips, 1993; Reese, Miller, Mazzeo, & Dossey, 1997). According to the recent Third International Mathematics and Science Study mathematics results, although U.S. students are a bit above the international average at 4th grade, by

8th grade they are below it; and by 12th grade they are near the bottom in international rankings[1] (Beaton et al., 1996; NCES 1996, 1997, 1998). In fact, there is hardly anywhere in America that the majority of students are performing at internationally competitive levels (Hershberg, 2000; Stevenson, 1995). For example, in the suburbs of one major American city in which the median family income is 30 percent higher than the national average, while three-quarters of students are at or above the international standard for computation, only between one-fifth and one-third meet the international standard in mathematics problem solving, "the quintessential skill required for the new economy" (Hershberg, 2000, p. 34).

Consequences of Poor Achievement for Individuals

In the U.S., the current system of mathematics education is taking a toll on both individuals and the nation. For individuals, the effects are like a long-term hidden illness that gradually incapacitates its victims (Battista, 1994). Although virtually all students enter school mathematically healthy, enjoying mathematics as they solve problems in ways that make sense to them, most exit school apprehensive and unsure about doing all but the most trivial mathematical tasks. Mathematics anxiety is widespread. So rampant is innumeracy in the U.S. that there is little stigma attached to it. Many adults readily confess "I was never good at mathematics," as if displaying a badge of courage for enduring, what for them, was a painful and useless experience. In contrast, no one freely admits that they can't read.

Furthermore, research shows that relatively few U.S. students graduate from high school equipped to handle the diverse mathematical and quantitative tasks required in their work, further education, informed citizenship, parenting, or personal finance (Gal, 1997). A full 75% of Americans stop studying mathematics before they complete career or job prerequisites (NRC, 1989).

Consequences of Poor Achievement for Ohio and the Nation

For the nation, the effects of mathematical miseducation are frightening. Because almost 40 percent of 17-year-olds do not have the necessary mathematics skills for a production job in manufacturing, today's manufacturers often must hire college graduates for work that high school graduates should be able to perform (van der Ploeg, 2001). The business sector spends as much on remedial mathematics education for employees as is spent on mathematics education in schools, colleges, and universities combined. And 60% of college mathematics enrollments reteach what should have been learned in junior or senior high school (NRC, 1989). Moreover, the problem is getting worse. By the year 2005, it is estimated that employers will have to increase expenditures by more than $15 billion to match the $63 billion spent training professional and technical workers

[1] In mathematics, of the 21 countries for which we have scores, the U.S. was fifth from the bottom. Statistically speaking, the U.S. mean was significantly lower than the international mean. The U.S. mean was significantly lower than the mean for 13 countries and significantly higher than the mean for only 2 countries.

in 1991 (van der Ploeg, 2001). Already, numerous technical jobs go unfilled or are given to foreign nationals because U.S. citizens lack sufficient mathematical and technical expertise (USCNS, 2001). Even the most elementary mathematical knowledge is in short supply in the work force (Packer, 1997), and only one in five business owners with firms that have 20-99 employees described themselves as "very" or "extremely" satisfied with the pool of employees available to them (Triangle Coalition, 2000). In fact, as the number of high-tech jobs in the US has grown by 1 million since 1993, the number of college graduates in the fields of mathematics, computer science and engineering has declined 5 percent (Triangle Coalition, 2000). "We're seeing industry add jobs, but the people with the right, specialized skills just aren't there," comments Michaela Platzer, vice president of research and industry trends at the American Electronics Association (Triangle Coalition, 2000).

In summary, within the context of the growing importance of universal high quality mathematics education for our nation's health and prosperity, international comparisons of students' mathematics achievement should stir a sense of foreboding about the nation our children will inherit. And Ohio's economic well-being is in double jeopardy; not only must it compete internationally, but it must compete with other states. Supporting only an elite class of students in mathematics education is no longer acceptable. *It is high time for Ohio's schools to start universally developing students' mathematical capability rather than merely selecting it.* Proper mathematics education is essential for individuals and governments (cf., Schmidt, McKnight, Raizen, 1997). It is an essential component in creating literate and informed citizens—the foundation for a well-functioning democracy. It helps each individual reach his or her full potential, both in work and personal life. It is an essential component for Ohio's and America's economic prosperity within a globally competitive, increasingly technological environment.

SECTION 2: A VISION FOR MATHEMATICS EDUCATION IN OHIO

A Statewide Mathematics Education System

Implementing high quality, world class mathematics education in Ohio requires the development of a comprehensive and coordinated statewide mathematics education *system*. To understand how to make this system maximally effective, we must first consider its components and how it operates.

System Components

The state mathematics education system consists of four major components: K-12 classrooms, school districts, universities, and state education agencies. The classroom, which consists of pupils and a teacher, is the central component because it is where student learning—the ultimate goal of the educational process—occurs. The other three components, however, have major effects on classroom learning.

School districts, which consist of pupils, teachers, and classrooms, along with administrators and the organizational structures that hold schools systems together, institute policies that affect classroom learning. For instance, districts determine how many students are in each class, how students are grouped in classes, which teachers teach what subjects and students, which tests are used to measure student performance, how teachers are supported in the classroom and out, and so on.

Universities, which consist of colleges of education and academic content area departments, have three major effects on classroom learning. First, they are responsible for educating teachers; and teacher quality has a major effect on classroom learning. Second, because universities are a major "consumer" of K-12 school systems, universities strongly influence the goals of K-12 education. Third, university faculty play a major role in conducting research on student learning and procuring funding for teacher inservice programs.

Finally, state education agencies heavily influence classroom learning because they determine learning goals through high-stakes tests, high school graduation requirements, regulations for teacher licensing, general educational regulations, and many school funding policies.

In analyzing this system, we must also consider one essential element that is not formally part of the system but is indirectly embedded throughout it. This critical element is the public, along with its mechanism for determining policy, governmental bodies. The public consists of people, along with the special interest groups into which they align themselves—parents and nonparents, business and industry, and so on. The public affects education policy formally through the mechanism of government and informally through influence-peddling activities of special interest groups. To have an effective state mathematics education system, not only must the four components of the system be integrated into a coherent whole, but the operation of this system must be properly connected with and supported by the public/government element.

To function properly, Ohio's mathematics education system must contain appropriate lines of communication with the public/government sector, but it must not be subject to undue interference that threatens its scientific and professional integrity. That is, although it is important that all stakeholders in the educational enterprise have appropriate input on the goals of education, because members of the public, in general, are not experts in mathematics education, often their beliefs about proper instruction conflict with scientifically sound educational practices. Thus, mechanisms must be instituted that (a) protect the professional practice of education from undue influence by nonexperts, and (b) educate members of the public and members of relevant governmental bodies about the benefits of implementing scientifically sound, professionally recommended practices in mathematics education. This can best be accomplished by bringing together leaders from the various components of Ohio's mathematics education system and public/government sector so that they can cooperatively improve the system in a coordinated and responsible way. And that is exactly the task that the Ohio Mathematics and Science Coalition is attempting to achieve.

Education Standards

NCREL Highlight: "In poll after poll, the public strongly supports standards. However, polls tell us little about what the content of standards should be. 'Basics first' is a common refrain and one the public often equates with a call for standards. However, 'basics first' does not fit well with current research about teaching and learning." (Otto & van der Ploeg, 2001a, p. 1)

Although there is much talk about standards in education today, there is a need to clarify what standards are and how they are being used. This section attempts to clarify some of the issues surrounding educational standards.

According to the Oxford English Dictionary, a standard is "An authoritative or recognized exemplar of correctness, perfection, or some definite degree of any quality," or "The substance or thing which is chosen to afford the unit measure of any physical quantity." In thinking about educational standards, therefore, we must decide what things or qualities we wish to examine, what degrees of correctness or perfection we desire for these qualities, and how to measure amounts of the qualities.

As used in NCTM's PSSM, "Standards are descriptions of what mathematics instruction should enable students to know and do—statements of what is valued for school mathematics education." (NCTM, p. 6). In fact, when most people think about educational standards, they generally focus on standards for student learning (and quite often, they consider only minimum standards). However, it is clear from the following statement that the developers of PSSM had more than learning standards in mind: PSSM is intended to "set forth a comprehensive and coherent set of goals for mathematics for all students…that will orient curricular, teaching, and assessment efforts…serve as a resource for teachers, education leaders, and policymakers to use in examining and improving the quality of mathematics instructional programs; guide the development of curriculum frameworks, assessments, and instructional materials; stimulate ideas and ongoing conversations…about how best to help students gain a deep understanding of important mathematics." (p. 5). That is, even though these standards include learning standards, statements about teaching, curricula, and assessment are intended.

Taking a systemic approach to mathematics education, therefore, cannot be limited to learning standards. In fact, there are four critical system components for which standards must be developed: (a) student learning (including goals—both cognitive and behavioral—and levels of performance), (b) teaching, (c) curricula (including textbooks), and (d) assessment. Specifying one component does not necessarily determine the others. If we are going to describe the nature of a high quality educational *system*, we must consider interrelated, aligned standards for all four components. And for clarity's sake, we should always specify to which components our standards refer.

Learning Standards

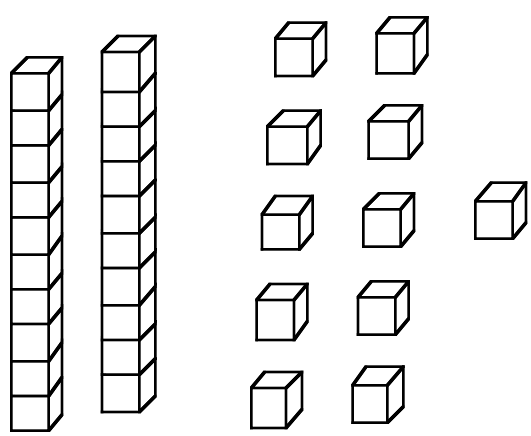
Because student learning must be the major focus of the mathematics education system, learning standards are critically important to specify. Mathematics learning standards should specify precisely the mathematical knowledge and competencies, as well as conceptualizations and ways of thinking and reasoning, that students should gain as a result of instruction. Learning standards should describe not only what formal mathematics students should know, but the exact nature of students' developing mathematical conceptualizations and ways of reasoning, as well as the particular age levels at which they should occur.

Consider, for example, the essential concept of place value, which can be understood at numerous levels of sophistication[2]. The NCTM 1989 Standards state that students in grades K-4 should "understand our numeration system by relating counting, grouping, and place value concepts." PSSM states that students in grades PreK-2 should "use multiple models to develop initial understandings of place value and the base-ten number system;" and in grades 3-5, "understand the place-value structure of the base-ten number system and be able to represent and

[2] The phrase "place value" refers to the concept that each digit in our numeration system signifies a value based on its place or position. For example, in the numeral 345, the value signified by the 3 is 300 because 3 is in the hundreds place, the value signified by the 4 is 40 because 4 is in the tens place.

compare whole numbers and decimals." Ohio's 1990 Model Competency-Based Mathematics Program states that students at grade 2 should "develop the concept of place value with concrete models of hundreds, tens, and ones" and "find equivalent forms of numbers using hundreds, tens, and ones;" students at grade 4 should "develop concepts of place value to include numbers through millions." And Ohio's draft Academic Content Standards for Mathematics state that at the end of the K-3 program, students should "use place value concepts to represent whole numbers using numerals, words, expanded notation, and physical models;" and at the end of the grade 4-5 program "use place value structure of the base-ten number system to read, write, represent and compare whole numbers and decimals." (Of course, use of the place-value concept is also embedded in many other objectives.)

None of these statements is precise enough, nor are they properly connected to research on students' mathematical cognition, to serve as a proper learning standard. For instance, supposedly consistent with all these standards, an activity in a second grade textbook commonly used in Ohio during the 1990's has an activity in which students are shown pictures of different configurations of tens and ones blocks and asked to write the number of tens and ones for each configuration. One configuration shows 2 tens rods and 11 ones and asks how many tens and ones.



tens _____ ones _____

A second-grader answered that there were 3 tens and 1 one. But her teacher told her this answer was wrong; that the answer was 2 tens and 11 ones. The student became confused and frustrated. And she had a right to be confused! There are several correct answers for this problem, even though only one was accepted by the text and teacher: 3 tens and 1 one, 2 tens and 11 ones, 1 ten and 21 ones, 0 tens and 31 ones.

Worse, research suggests that activities and standards like these that disembed place value from its use in more general numeric thinking may be too abstract for primary-age students to understand (Thompson, 2000). Instead, place-value understanding at the primary ages should manifest itself in children's invention and use of ways to add and subtract multidigit numbers. Children seem to naturally progress from adding numbers using a counting-by-ones strategy to a counting-by-tens-and-ones strategy or the strategy of collecting-tens-and-ones (Fuson et al. 1997). For instance, to add 35 and 27, students who are supported by appropriate instruction naturally progress to an abbreviated form of counting that incorporates an implicit understanding of place value—35, 45, 55, 56, 57, 58, 59, 60, 61, 62; or they might say 30 and 20 makes 50, 5 and 7 makes 12, so the answer is 50 + 12 = 62. So, an appropriate learning standard for place value at the primary grades might be that by the end of second grade, students possess an implicit understanding of place value that enables them to add and subtract 2-digit numbers using the counting-by-tens-and-ones and the collecting-tens-and-ones strategies.

Forcing students to use the standard addition algorithm—or to think abstractly in terms of tens or ones in preparation for use of that algorithm (as in the above "tens-ones" activity)—can cause students to bypass the critical process of constructing meaning for these ideas from counting. (And using this algorithm does not indicate understanding of place value because most students learn it rotely.) Because meaning for numeric ideas must be grounded in counting, the overly abstract view of place value and numbers promoted in traditional texts and many mathematics learning standards can derail students' construction of genuine meaning for these critical ideas.

The place-value example illustrates that specification of mathematics learning standards is extremely complex. It cannot be accomplished without careful analysis of (a) scientific research on students' learning for particular ideas, and (b) detailed descriptions of students' mathematical cognitions. Because current Ohio and national standards are too general and are insufficiently based on research, they are inadequate for guiding teaching, curriculum, and assessment. We need more precise, research-based, and carefully thought-out standards. (Ohio and national standards are valuable in that they indicate, in a general way, the mathematical content that professionals agree are important for students to learn. But they do not specify the nature of student learning that must occur for students to acquire that content, and this is what is needed to guide instruction.)

Different Perspectives

Levels of Engagement of Students in Mathematics Learning

When different people or groups within the mathematics education system consider learning standards, goals, assessment, and success, they often focus on very different aspects of students' participation in school instruction. The "levels of engagement" scale below is useful for understanding the various approaches, goals, and perspectives that have been taken in assessing the effectiveness of mathematics instruction and curricula.

Level 0: Students disengaged from the intellectual life of school

Perhaps because of a mismatch between the goals and culture of the school with those of particular students, students "drop out" of the intellectual activities in school and involve themselves only in its social aspects.

Level 1: Students engaged in the intellectual life of school but disengaged from *mathematics* learning

Students attempt to involve themselves in academic aspects of school. But, perhaps because of past failures, or because students see no relevance of mathematics to their lives, students decide that doing well in, or even enrolling in, mathematics courses is not important to their lives.

Level 2: Students engaged in learning mathematics as memorization and mimicry, but disengaged from mathematical sense making

Students do not find intrinsic value or gratification in learning mathematics. But because they have embraced the overall academic values of school, they still try to get good grades and enroll in appropriate mathematics classes. However, because traditional instruction has made personal sense making inaccessible for them, these students resort to memorization and mimicry as the primary focus of learning.

Level 3: Students engaged in learning mathematics as sense making

Students attempt to make personal, physical, quantitative, and abstract sense of mathematics. They not only find extrinsic, career-oriented value, but intrinsic value and gratification in learning mathematics.

Implications

An effective mathematics education system must consider all four levels on this scale. Although the first level does not fall exclusively within the purview of mathematics education, both it and the second level can be addressed by making mathematics more interesting and relevant to students. One way to do this is for instruction to focus on real-world applications. But at a fundamentally deeper level, instruction must focus on mathematical sense making and understanding. That is, students will be more interested and persistent in mathematics courses and curricula that enable them to be successful in personal sense making. Students who are given opportunities to be successful at understanding mathematics throughout their education will experience the internal rewards that keep them meaningfully engaged in mathematics learning. Students who fail to genuinely understand mathematics and must resort to rote memorization will feel little intrinsic satisfaction and are likely to withdraw from mathematics learning. "Understanding breeds confidence and engagement; not understanding leads to disillusionment and disengagement" (Hiebert et al., 1997, p. 2). Thus, *focusing on personal sense making and understanding is the key to the overall success of a mathematics education program.* Instructional programs with such a focus not only increase the quality of students' mathematics learning, but the quantity of mathematics taken (COMPASS, 2000).

The scale also helps us understand different approaches to improving and evaluating the success of mathematics programs. For instance, many programs focus on moving students from Levels 0 and 1 to Level 2. As a measure of success, they typically examine trends in mathematics course enrollment or completion data. Obviously, increasing student enrollment in appropriate mathematics classes is valuable. However, many programs with this goal fail to consider the core role that personal sense making plays in students' successful mathematics learning. For instance, there are numerous calls for all students to complete high school algebra. But simply requiring students to take traditional high school algebra, a course which research shows is extremely ineffective, is unlikely to provide genuine long-term solutions to the problem. In fact, all that may happen is that as more students enroll in traditionally ineffective courses, failure rates will skyrocket (or course standards will decline). As another example, increased mathematics requirements for high school graduation are praiseworthy. But if the courses offered to students do not appropriately support their mathematical sense making and understanding, the additional courses may have little long term effects on student learning (Bell, 1989) and, instead, increase mathematics anxiety among students.

In summary, while various techniques for increasing enrollments and participation in mathematics courses can sometimes be valuable, genuine improvement in students’ mathematics learning can only be accomplished through scientifically sound instructional techniques that elicit and support students' personal mathematical sense making and genuine understanding.

The Role of Scientific Research in Educational Practice

Today, the world is in the midst of an extraordinary outpouring of scientific work on the mind and brain, on the processes of thinking and learning, on the neural processes that occur during thought and learning, and on the development of competence. …A new theory of learning is coming into focus that leads to very different approaches to the design of curriculum, teaching, and assessment than those often found in schools today. (Bransford, Brown, & Cocking, 1999).

Although they have been profoundly challenged by the past three decades of research in cognitive science and related disciplines, the assumptions [about learning and teaching] of the 1920s are firmly ensconced in the standard operating of today's schools. (Resnick & Hall, 1998).

One of the major reasons that American educational practice in general, and school mathematics programs in particular, make so little progress in improving students' learning is because they ignore scientific research findings and fail, in their practice, to adhere to scientific methodology. That is, most educational programs and methods used in schools are formulated with a total disregard for modern scientific research on mathematics learning. Because educational practice is neither properly grounded in scientific research nor subject to the critical scrutiny of scientific analysis and review, educators continually "reinvent the wheel," often following one bandwagon after another. In fact, Wilson and Daviss (1994) liken the current state of educational curriculum development in the United States to that of aeronautics before the time of the Wright brothers at Kitty Hawk.

A century ago, people making airplanes were usually solitary, self-taught visionaries or eccentrics following their own theories or hunches. They lacked a good deal of information about aerodynamics...They continued to work separately, often unknowingly crossing and recrossing each other's tracks, unable to take advantage of or build on each other's successes. (Wilson and Daviss, 1994)

To steer Ohio's mathematics education system toward significant improvement of student learning, educational practice must be based on and conducted with methods from scientific research about how students learn. Because scientific knowledge is constructed according to rigorous standards of reasoning and verification upheld by scholars who constantly review, test, critique, and build on each others' work, scientific knowledge is more reliable than the commonsense ideas and folk wisdom most people use to make judgments about teaching and learning. Moreover, because most educated members of our culture hold scientific knowledge in high esteem, relying on scientific knowledge and analysis can serve as a focal point for consensus building in rational discussions of educational practice.

Certainly, however, research alone cannot specify mathematics curricula; much of what is taught is determined by the goals decided upon by various stakeholders in the educational system. Nevertheless, research can tell us what goals are achievable, which students are attaining goals, how various goals can be achieved, and how different factors affect goal attainment. Thus, while it is appropriate for Ohio's mathematics education to consider public input, it is absolutely critical that the system's practices and policies be firmly grounded in scientific research and analysis.

A Scientifically Sound and Professionally Recommended Approach to Teaching and Curriculum

What Scientific Research Says about Mathematics Learning

NCTM Recommendation: *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge….In recent decades, psychological and educational research on the learning of complex subjects such as mathematics has solidly established the important role of conceptual understanding in the knowledge and activity of persons who are proficient. (NCTM, 2000, p. 20)*

What Does It Mean to Understand Mathematics

The central goal in NCTM's view of mathematics education is for students to make sense of mathematics, to understand it. But *understanding* is a difficult concept to characterize precisely. Hiebert et al. (1997) state that we understand something when we see how it is related or connected to other things, and that understanding comes about as we reflect and communicate. In fact, knowledge is understood and "usable" not when it consists of a mere list of disconnected facts, but when it is connected and organized around important concepts (Bransford, Brown, & Cocking, 1999). Johnson-Laird extends the description of understanding: “If you know what causes a phenomenon, what results from it, how to influence, control, initiate, or prevent it, how it relates to other states of affairs or how it resembles them, how to predict its onset and course, what its internal or underlying ‘structure’ is, then to some extent you understand it" (1983).

Johnson-Laird and many other researchers claim that the psychological core of understanding consists in having a working mental model of the phenomenon in your mind. Mental models are nonverbal recall-of-experience-like mental versions of situations that have structures identical to the perceived structures of the situations they represent. They are abstracted from experiences and reflection on those experiences, and most often have an image-like quality. When using a mental model to reason about a situation, a person can mentally move around, move on or into, manipulate, combine, and transform objects, as well as perform other operations like those that can be performed on objects in the physical world. Thus, genuine mathematical understanding and personal sense making must always be grounded in appropriate mental models for mathematical phenomena.

Personal Sense Making Is Critical to Learning with Understanding

All current major scientific theories describing how students learn mathematics with genuine understanding (instead of by rote) agree that (a) mathematical ideas must be mentally constructed by students as they intentionally try to make personal sense of situations, (b) how students construct new ideas is heavily dependent on the cognitive structures students have previously developed, and (c) to be effective, mathematics teaching must carefully guide and support the processes by which students construct mathematical ideas (Battista & Larson, 1994; Bransford, Brown, & Cocking, 1999; Cognition and Technology Group at Vanderbilt, 1993; De Corte, Greer, & Verschaffel, 1996; Goldin, 1992; Greeno, Collins, & Resnick, 1996; Lesh & Lamon, 1992; NRC, 1989; Lester, 1994; Hiebert & Carpenter, 1992; Mack, 1990; McCombs, 1993; Prawat, 1999; Resnick, 1995; Romberg, 1992; Schoenfeld, 1994; Steffe & Kieren,1994). According to this scientific theory and research, the way a student interprets, thinks about, and makes sense of newly encountered mathematical ideas is determined by the elements and organization of the set of relevant mental structures that the student has previously constructed and is currently using to process his or her mathematical world. Consequently, instruction that promotes understanding cannot ignore students’ current ideas and ways of reasoning, including their many informal, even incorrect, ideas. "If students' initial ideas and beliefs are ignored, the understandings that they develop can be very different from what the teacher intends" (Bransford, et al., 1999, p. 10).

Thus, mathematics instruction that produces conceptual understanding rather than rote memorization, and genuine problem solving ability rather than "mindless mimicry," must be based on careful and detailed analyses of (a) students' current concepts and ways of reasoning, (b) the mental processes by which students' construct new mathematical knowledge and (c) the actual concepts and ways of reasoning students construct as they attempt to learn particular mathematical ideas.

Organization of Knowledge Is Critical

Another critical finding of modern research on learning is that mathematical knowledge is truly understood and usable when it is organized around and interconnected with important core concepts, not when it consists of lists of disconnected facts (Bransford, Brown, & Cocking, 1999; Hiebert et al., 1997). Unfortunately, traditional mathematics curricula, like many other school curricula, "make it difficult for students to organize knowledge meaningfully. Often there is only superficial coverage of facts before moving on to the next topic; there is little time to develop important, organizing ideas" (Bransford, Brown, & Cocking, 1999). Indeed, one of the major findings of TIMSS is that U.S. mathematics curricula cover too many topics, so most students learn them at a surface level, at best (Schmidt, McKnight, & Raizen, 1997). That is, while nations that do better than the U.S. teach 5 to 7 "fundamental concepts" per year, American schools attempt to teach 25 to 30, resulting in superficial, poorly sequenced instruction that TIMSS characterized as "a mile wide and inch deep." Though the same topics are taught and retaught year after year in American schools, many are never learned, few are truly understood.

Similarly, despite research indicating that learning with understanding, instead of learning by memorization, produces better ability to transfer knowledge so that it can be applied in new situations (Bransford, Brown, & Cocking, 1999; Mayer & Wittrock, 1996), traditional American instruction places more emphasis on memorization and imitation than on understanding, thinking, and reasoning (De Corte, Greer, & Verschaffel, 1996; Greeno, Collins, & Resnick, 1996; Silver & Stein, 1996). That is, traditional instruction focuses on what is easy to teach and assess rather than what students need to learn.

NCTM-Based Recommendations for Curricula and Instruction

In response to (a) the documented failure of traditional methods of teaching mathematics, (b) curriculum changes necessitated by the widespread availability of computing devices, and (c) a major paradigm shift in the scientific study of mathematics learning, the movement to revamp and improve school mathematics instruction began in the mid-1980s. The most conspicuous component of the improvement efforts has been the attempt by schools and teachers to implement the recommendations given in the "standards" for school mathematics published by the National Council of Teachers of Mathematics (NCTM) (1989, 2000). These recommendations deal with how mathematics is taught, what mathematics is taught, the nature of school mathematics, and who should learn mathematics. (A summary of NCTM's latest "principles" and "standards" for school mathematics is given in Appendix A.)

How Mathematics Is Taught: Teaching Sense Making and Problem Solving

The focus in the classroom environments envisioned by NCTM is on inquiry, sense making, and problem solving. In such classrooms, teachers provide students with numerous opportunities to solve complex and interesting problems; to read, represent, discuss, and communicate mathematics; and to formulate and test the validity of their mathematical ideas using demonstrations, drawings, and physical objects—as well as formal mathematical and logical arguments. As emphasized in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), teachers "need to create an environment that encourages children to explore, develop, test, discuss, and apply ideas. They need to listen carefully to children and to guide the development of their ideas" (p. 17).

Research-Guided Inquiry-Based Teaching

One of the major techniques that sense-making instruction uses to nurture students' construction of meaning of mathematical ideas is inquiry-based instruction. According to the National Academy of Science (NSF, 1997), inquiry is a multifaceted activity that involves making observations; posing questions; examining sources of information to see what is already known; planning and conducting investigations; gathering, analyzing, and interpreting data; proposing and testing solutions, explanations, and predictions; arguing and debating; and communicating the results for others to scrutinize. Inquiry teaching supports students' natural inquisitiveness and sense making. It involves students in personal sense making and knowledge construction in a culture of collaborative involvement with problem-solving activities. As students reflect on ideas that they generate in attempting to solve problems, and as they communicate their ideas to others and reflect on ideas communicated by others, they construct their own personal versions of those ideas based on their already existing relevant knowledge.

But truly effective inquiry-based teaching must be research-based (Battista, in press). That is, maximally effective inquiry-based instruction (a) is based on careful analysis of research on students' learning of particular mathematical ideas, and (b) poses appropriate sequences of tasks and engages students in appropriate discussions of those tasks to carefully guide students' *personal construction* of mathematical ideas. Research-guided inquiry-based instruction stimulates and supports students' development of personal mathematical meanings that are more complex, abstract, and powerful than they currently possess.

Note that lecture and demonstration, long-cherished tools in traditional instruction, can promote meaningful mathematics learning in students only if two conditions hold. First, students must be mathematical sense makers (i.e., they must believe that they can make sense of mathematics, be committed to making such sense, and have developed sense making skills). Second, students must have the intellectual wherewithal to make personal sense out of the mathematics presented (i.e., the material presented is within students' current "zones of construction"). Neither condition is likely to exist if students have not previously been involved in significant amounts of genuine research-guided inquiry-based mathematics instruction.

NCREL Highlight: Results from a Conference with Board Certified Ohio Teachers. Teachers believe that classroom instruction should:

- consist of in-depth explorations of fewer topic areas
- consist of real-world contexts and connections to life experiences
- consist of project and inquiry-based activities
- shift from teacher talk to group-centered work
- provide writing, speaking opportunities for student self expression
- build on student knowledge
- model a passion for mathematics (Otto, van der Ploeg, & Blakeslee, 2000)

What Mathematics Is Taught: Focusing on Basic Skills of Today, Not of 40 Years Ago

In traditional mathematics instruction, students spend most of their time attempting to learn traditional paper-and-pencil computational procedures—procedures that can now be implemented far more quickly and accurately on a calculator. Few students develop any substantive understanding of why computations work or, more importantly, when they should be applied.

In mathematics curricula recommended by NCTM and all other professional organizations that deal with mathematics education, the exclusive emphasis that traditional teaching places on paper-and-pencil computation is moderated. Increased attention is given to mathematical reasoning and problem solving as well as previously-neglected topics such as statistics and the use of computational devices in mathematical analysis. These curricula focus on the basic skills of today, not those of 20 or 40 years ago. Problem solving, reasoning, justifying ideas, making sense of complex situations, and independently learning new ideas—not paper-and-pencil computation—are now critical skills for all Americans. In the information-and-web era, obtaining information is not the problem; analyzing and making sense of it is.

UC Ohio Survey Highlight. Almost all respondents identified basic arithmetic (98%) as necessary for all students before graduation from high school. A high percentage (95%) identified reasoning and problem-solving as important; 72% identified algebra as important. Knowing how to communicate and explain mathematics (75%) and using calculators and computers to do mathematics (71%) were also identified as important. (The later two figures show that there are still substantial numbers of Ohioans who need further education on the nature of mathematics needed by school children.)

Only 56% of respondents identified geometry as important, indicating that this topic, which is internationally recognized among mathematics educators as important, is not being taught in a way that engenders appropriate learning and appreciation among students.

The Nature of School Mathematics: Seeing Mathematics As Reasoning Rather than Rule-Following

Mathematics is first and foremost a form of reasoning. In the context of analytically reasoning about particular types of quantitative and spatial phenomena, mathematics consists of thinking in a logical manner; formulating and testing conjectures; making sense of things; as well as forming and justifying judgments, inferences, and conclusions. We do mathematics when we recognize and describe patterns; construct physical as well as conceptual models of phenomena; create symbol systems to help us represent, manipulate, and reflect on ideas; and invent procedures to solve problems.

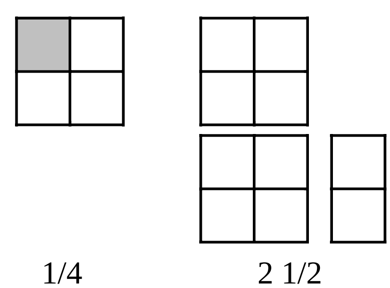
Meaningful Mathematics Learning versus Parroting Procedures

To illustrate the difference between instruction that emphasizes personal sense making and instruction that emphasizes computation, consider the problem, "What is $2\,{}^1\!/_2$ divided by ${}^1\!/_4$?" Traditionally-taught students are trained to solve such problems by using the "invert and multiply" method (which students memorize, quickly forget, and almost never understand):

$$2\,{}^1\!/_2 \div {}^1\!/_4 = {}^5\!/_2 - {}^4\!/_1$$

In the traditional, computation focused curriculum, those students who are lucky enough to be able to compute an answer to this problem rarely can explain or demonstrate why the answer is correct (other than saying something like "My teacher said we are supposed to invert and multiply"). But worse, students rarely know when the computation should be applied in real-world contexts (e.g., Kouba, Zawojewski, & Strutchens, 1997; Ma, 1999).

In contrast, in a sense-making curriculum, students do not need a symbolic algorithm to compute an answer to this problem. Because they interpret the symbolic statement in terms of appropriate mental models of quantities, they are quickly able to reason that, since there are 4 fourths in each unit and 2 fourths in a half, there are 10 fourths in $2\,{}^1\!/_2$. (Younger students might draw a picture to support such reasoning.) Furthermore, such students are able to quickly recognize when to apply such thinking in real-world situations.



Students in sense-making mathematics curricula are not manipulating symbols, oblivious to what the symbols should represent, but are purposefully and meaningfully reasoning about quantities. They are not blindly following rules invented by others, but are making personal sense of ideas. These students develop powerful conceptual structures and reasoning patterns that enable them to apply their mathematical knowledge to numerous real-world situations, giving them intellectual power and autonomy in their mathematical reasoning.

Obviously, not all problems can be easily solved using intuitively appealing strategies such as that demonstrated with this fraction problem. Students must also develop understanding of and facility with symbolic manipulations, and even an appreciation for the workings of axiomatic systems that describe how to deal formally with mathematical symbols. That is, it is not enough to involve students only in sense making, reasoning, and the creation of new mathematical knowledge. Sound curricula must have clear long-range goals for assuring that students become *fluent* in utilizing those abstract concepts and mathematical perspectives that our culture has found most useful. They should possess appropriate knowledge that supports mathematical reasoning. For instance, students should know the “basic number facts” because such knowledge is essential for mental computation, estimation, computation with multidigit numbers, and problem solving.

Nonetheless, students' learning of symbolic manipulations must never become disconnected from their reasoning about real-world quantities. For when it does, they become overwhelmed with trying to memorize countless rules for manipulating symbols. Even more, when students lose sight of what symbol manipulations imply about real-world quantities, doing mathematics becomes an academic ritual that has no real-world usefulness. Indeed, to be able to use mathematics to make sense of the world, students must first make sense of mathematics.

Differences between school mathematics and mathematics as a discipline

Significantly, the view of mathematics that emerges in reform classrooms is similar to how many mathematicians describe the process of doing mathematics, as opposed to how the results of mathematical activity are formally recorded in books and journals. For instance, mathematician Saunders MacLane's states: "Mathematics starts from a variety of human activities, disentangles from them a number of notions which are generic and not arbitrary, then formalizes these notions and their manifold interrelations" (as quoted in Pinker, 1997, p. 340). George Polya observes: "Finished mathematics presented in a finished form appears as purely demonstrative….. Yet mathematics in the making resembles any other human knowledge in the making. You have to guess a mathematical theorem before you prove it; you have to have the idea of the proof before you carry through the details. You have to combine observations and follow analogies; you have to try and try again" (as quoted in Hersh, 1997, p. 36). Finally, Reuben Hersh says, "In developing and understanding a [mathematical] subject, axioms come late. Then in the formal presentations, they come early….The view that mathematics is in essence derivations from axioms is backward. In fact, it is wrong…..The standard exposition purges mathematics of the personal, the controversial, the tentative, leaving little trace of humanity in the creator or consumer" (1997, p. 6). The picture of mathematics that traditionally taught students develop is a mere caricature of genuine mathematical activity.

In Schools	As a Discipline
Mathematics is neat and concise. It is about memorizing correct procedures or algorithms for solving well-defined problems.	Mathematics is messy. It involves a search for sense and order from complex, ill-defined situations.
Speed and correct answers are emphasized.	Persistence and flexibility are essential to mathematical pursuits. Mathematicians often spend years trying to solve a single problem.
Answers are validated by the teacher or answer book.	There is no answer book. Often there are no "best" answers or even a guarantee that an answer will be found.
Calculators may be used only once basic skills are mastered. Computers and other technologies are useful primarily for drill but also for enrichment.	Tools (manipulatives, computers, calculators) are continuously used to examine and represent ideas or extend thinking. Tedious computations are done by machines. Thinking and reasoning are done by people.
Math is done in isolation, working quietly from a textbook or a worksheet.	Math is a collaborative endeavor with mathematicians and others working together, communicating their ideas and building on one another's ideas and experiences.

(Otto & van der Ploeg, 2001b, p. 2)

<u>NCREL Highlight:</u> Results from a Conference with Nationally Board-Certified Ohio Teachers (Raw data, NCREL). Teachers believe that students should:

- be independent, critical, analytical thinkers
- be intellectual, creative, innovative risk takers
- be managers in addition to seekers of information
- be problem solvers
- be self-confident, passionate, socially conscious
- understand mathematics rather than rotely manipulate formulas
- see concepts rather than correct solutions
- make connections within and beyond their knowledge
- work collaboratively, communicate effectively, be accepting of, and discuss different approaches
- present sound arguments
- understand mathematics as a way of thought rather than a set of formulas and definitions
- use mathematics concepts as tools to solve problems rather than as objects to memorize
- apply mathematics to real-world problems

What Scientific Research Says about High Quality Mathematics Teaching

NCTM Recommendation: *Effective mathematics teaching requires a serious commitment to the development of students' understanding of mathematics. Because students learn by connecting new ideas to prior knowledge, teachers must understand what their students already know. Effective teachers know how to ask questions and plan lessons that reveal students' prior knowledge; they can then design experiences and lessons that respond to, and build on, this knowledge. (NCTM, 2000, p. 17)*

Fluency in mathematical problem solving and reasoning is a critical basic skill in the modern world and must be supported for all students, not just the privileged few, as has happened traditionally. Research shows that the type of mathematics learning that supports such fluency is based on deep conceptual understanding of mathematical ideas (Carpenter & Lehrer, 1999). And to produce such deep conceptual understanding of mathematics in students, curricula and instruction must be firmly grounded in a firm understanding of students' mathematical cognition, the type produced by modern research in mathematics education.

Good Teaching Guides Students' Personal Construction of Mathematical Ideas

To develop powerful mathematical thinking in students, instruction must focus on, guide, and support students' personal construction of meaning for ideas. In contrast to traditional instruction's tight focus on students parroting standard formal mathematical procedures, meaning-centered instruction focuses on the ever-changing and complicated ideas that students construct as they struggle to make personal sense of mathematical ideas. Such instruction attempts to understand how students' understanding is developing and to use that knowledge to carefully nurture the development of students' thinking so that it evolves to the point where students can competently and comprehensibly utilize formal reasoning and techniques. Such instruction encourages students to formulate, test, and refine their own ideas rather than thoughtlessly follow procedures given to them by others.

For example, to be genuinely meaningful, students' thinking about symbolic procedures for manipulating fractions must be appropriately connected to their thinking about physical fractional quantities. That is, students make symbolic manipulations of fractions personally meaningful as they are carefully guided to connect these manipulations to their experiences dealing with physical fractional quantities. In this way, students' knowledge of symbolic procedures becomes grounded in and connected to their rich reasoning about physical quantities.

Teaching Must Be Continuously Guided by Teachers' Understanding of Students' Current Knowledge and Reasoning

A consequence of the view that new knowledge must be constructed from existing knowledge is that instruction must explicitly attend to the informal ideas, naïve concepts, incomplete/incorrect understandings, and false beliefs that students bring with them when learning new mathematics. Teachers must find ways to help students build on their current ideas in ways that enable each student to achieve more mature understandings. "There is a good deal of evidence that learning is enhanced when teachers pay attention to the knowledge and beliefs that learners bring to a learning task, use this knowledge as a starting point for new instruction, and monitor students' changing conceptions as instruction proceeds" (Bransford, Brown, & Cocking, 1999).

Thus, to be truly effective, teachers must make instructional decisions based on knowledge from scientific research on how students learn particular mathematical content. They must have an understanding of the stages that students pass through in constructing meaning for particular mathematical concepts and procedures, the strategies students use to solve problems at each stage, and the nature of the mental processes that support students' developing competence with these new mathematical topics (Carpenter & Fennema, 1991). **S**election of instructional tasks must be "grounded in detailed analyses of children's mathematical experiences and the processes by which they construct mathematical knowledge" (Cobb, Wood, & Yackel, 1990, p. 130). Without appropriate attention to and knowledge of student meanings and cognitive constructions —knowledge generated by modern research in mathematics education—teachers and curriculum developers cannot be successful in properly educating all students.

Scientific Support for Inquiry-Based Instruction that Is Guided by Research on Students' Mathematical Thinking

An abundance of research has shown that reform-based mathematics instruction that focuses on inquiry, problem solving, and personal sense making produces powerful mathematical thinkers who not only can compute but have strong conceptions of mathematics and problem-solving skills (Hiebert, 1999). For example, in a year-long project conducted by Cobb et al. (1991), 10 second-grade classes that participated in mathematics instruction that was compatible with the tenets described above were compared to 8 traditional classes. At the end of the study, the levels of computational performance of the two groups were comparable, but students in reform-based instruction had higher levels of conceptual understanding in mathematics; held stronger beliefs about the importance of understanding and collaborating; and attributed less importance to conforming to the solution methods of others. Similarly, Wood and Sellers (1996, 1997) compared classes receiving problem-centered mathematics instruction in second and third grade for two years to traditional classes. Results indicated that in reform-based classes, students scored significantly higher on standardized measures of computational proficiency as well as on conceptual understanding. Research by Muthukrishna and Borkowski (1996) found that the meaningful mathematics learning performance of third graders in reform-based instruction not only was superior to that of students in direct instruction and traditional instruction, but that the reform-based students evidenced greater use of "deep processing strategies."

Similar positive results for inquiry- and problem-solving-based, sense making instruction were obtained by Fennema et al. (1996) and Carpenter et al. (1998) at the primary grade level; Ben-Haim et al. (1998) and Silver et al. (1996) at the junior high level, with the latter in urban districts, and Quinn at the university level (Quinn, 1997). At the senior high level in England, Boaler (1998) found that "Students who followed a traditional approach developed a procedural knowledge that was of limited use to them in unfamiliar situations. Students who learned mathematics in an open, project-based environment developed a conceptual understanding that provided them with advantages in a range of assessments and situations."

Other Curricular Concerns

Equity—Making Teaching Appropriate for ALL Students, Not Just the Elite

One of the major thrusts of the reform movement in mathematics education is the push to provide a high-quality mathematics education for ALL students, including minority as well as majority students, females as well as males, those who are going to college or into scientific fields and those who are not. In the traditional approach to mathematics instruction, only the very top students, most from the cultural majority, have been served (and even their learning has not been optimized).

In equitable educational programs, the goal is for all students to learn with understanding. The unwritten but common traditional notion that only smart students can learn with understanding must be purged. Research indicates that, with appropriate instruction, all students can learn with understanding (Hiebert et al., 1997). Indeed, the attitude that all students can succeed if they work hard enough and are taught properly is part of the educational philosophy in many countries in which school students' mathematics achievement greatly exceeds that in the United States (Stevenson, 1995).

Making Mathematics Instructionally Relevant, But Not Vocational

In attempts to make mathematics education more inclusive, there are calls to make mathematics more relevant to students' lives. It is argued that teaching academic skills in the context of realistic applications provides motivational benefits, imparts deeper understanding, increases retention, and enhances students' ability to apply their academic skills (Bailey, 1998). But in moving towards relevance, instruction should not become vocation specific, narrow, or focused on low level skills (Hoachlander, 1997, p 124-5). For instance, "Most advanced high school mathematics has rigorous, interesting applications in the work world" (Hoachlander, 1997, p 135). In fact, many high-level thinking goals in the NCTM *Standards* are essential in the work world. For instance, Fessenden argues that the ability "to inhabit simultaneously the business world and the mathematical world, to translate between the two, and, as a consequence, to bring clarity to complex, real-world issues is of extraordinary importance" (1998, p. 21). This process is the heart of mathematical modeling. Another critical skill needed in business is the ability to listen to a presentation and evaluate which results are reliable, which are subject to reinterpretation, which are judgments not supported by appropriate analysis, and which are hypotheses that require further research (Fessenden). These are precisely the skills promoted in the inquiry-based mathematics classrooms recommended by reform (and almost totally ignored in traditional mathematics instruction).

Preparing All Students to Enter College

Another major push in the equity component of reform is for high schools to prepare all students for college, in particular, getting all students to remain in college-appropriate mathematics courses. One reason for doing this is to keep students' options open as they embark on their adult lives. For instance, Taylor claims that algebra, the traditional gateway to higher mathematics, is important for all students: "Algebra is also important for those students who do not currently aspire to mathematics-based careers, in part because a lack of algebraic skills puts an upper bound on the types of careers to which a student can aspire" (1998, p. 31). Moreover, to achieve an adequate level of numeracy in today's fast-changing, information/technology intensive world, school mathematics instruction should not only make students technically competent, it should make them comfortable enough with mathematics so that they will later be willing to invest in further mathematics-based learning when they need it (Gal, 1997).

The other reason for encouraging all students to remain in college appropriate curricula is the wage gap between college and high school graduates: "During the last twenty years, the real wages of high school graduates have fallen and the gap between the wages earned by high school and college graduates has grown significantly. Adults with no education beyond high school have very little chance of earning enough money to support a family with a moderate lifestyle. Given these wage trends, it seems appropriate and just that every high school student at least be prepared for college, even if some choose to work immediately after high school" (Bailey, 1998, p. 26).

Properly Educated and Supported Teachers

It has been found repeatedly that reforms are rendered effective or ineffective by the knowledge, skills, and commitments of teachers and principals (Hammond & Ball, 1997). Thus, school systems must select, prepare, and retain teachers and principals who understand and can implement a modern, scientifically based perspective on teaching and learning of mathematics.

Teacher Knowledge of Mathematics and of Students' Mathematical Thinking Is Critical in Supporting High Student Achievement

A number of studies have shown that teacher expertise is an important factor in determining student achievement (Hammond & Ball, 1997). What teachers understand about mathematics, student learning, and teaching shapes their instructional methods and interactions with students, their selection of instructional materials, their assessment of students' learning, and, indeed, their teaching effectiveness.

Knowledge of mathematics

There is an overall consensus among teacher educators and researchers that the education of teachers needs to focus much more attention on basic mathematical reasoning (Ball, in press;

MSEB, 1996; Lappan & Even, 1994; Ma, 1999). In fact, "Recent research highlights the critical influence of teachers' subject matter understanding on their pedagogical orientations and decisions. . . . Teachers' capacity to pose questions, select tasks, evaluate their pupil's understanding, and make curricular choices all depend on how they themselves understand the subject matter" (McDiarmid, Ball, and Anderson, in press).

Preparing future teachers to be effective in implementing reform-based mathematics instruction requires teachers to develop an understanding of mathematics that is far deeper, broader, and richer than most teachers currently possess. (Schifter & Bastable, 1995). This understanding must have both depth (organizing knowledge around conceptually central ideas) and breadth (being able to connect topics to similar ideas, knowing how a topic develops in sophistication as grade level increases). It must enable teachers to see how school mathematics concepts are connected to both concrete embodiments and formal abstract mathematical representations. Unfortunately, there is currently a vicious cycle in which too many future teachers enter college with serious holes in their understanding of school mathematics, have little college instruction focused on the mathematics they will teach, and so enter their classrooms inadequately prepared to teach mathematics (Battista, 1994).

Pedagogical knowledge: Knowing students' mathematical thinking and appropriate instructional methods

However, care must be taken in focusing on teachers' knowledge of mathematics. Even when teachers are taught additional mathematical content, little improvement in teaching may result. For instruction to improve, teachers must learn content in a way that applies to their teaching (MSEB, 1996). In fact, to be effective, teachers need more than a firm understanding of mathematics; they must also understand how students construct understandings of specific mathematical topics and they must be able to capably employ instructional methods that support and optimize these the constructive processes. Indeed, research shows that, “the extent of teachers' preparation in mathematics methods, curriculum, and teaching is as important in predicting effectiveness as is preparation in mathematics itself" (Hammond & Ball, 1997). “Teachers who have spent more time studying teaching are more effective overall, and strikingly so for developing higher order thinking skills and for meeting the needs of diverse students" (Hammond & Ball, 1997). Especially when it comes to teaching for understanding, teachers' mathematical content knowledge and their knowledge of students' mathematical cognition are critical.

Strong Preservice Education

Consistent with the knowledge requirements for teachers described above, graduates of teacher education programs must have firm knowledge of mathematics, how students learn mathematics, and effective instructional methods for producing mathematics learning. Mathematics content and methods courses must not be squeezed out of the curriculum by courses in general pedagogical preparation. Connections must be made between what is taught in mathematics content courses, methods courses, and what future teachers will teach. Student teaching experiences must be arranged with teachers who are skilled in scientifically sound, professionally recommended practices for teaching mathematics (as opposed to traditional methods).

Preservice teachers must be taught mathematics properly before we can expect them to teach it properly. Universities must take the lead in making changes in the way mathematics is taught (Battista, 1994). Teacher education institutions need to offer numerous mathematics courses for teachers that treat mathematics as sense making, not rule-following. Mathematics courses should be taught through inquiry-based methods and should promote future teachers' personal sense making so that they experience as students the kind of high quality mathematics teaching we expect them to deliver to their pupils. As the National Research Council states, “Teachers themselves need experience in doing mathematics—in exploring, guessing, testing, estimating, arguing, and proving...[they] should learn mathematics in a manner that encourages active engagement with mathematical ideas” (1989, pp. 65-66).

Systemic Support for Teacher Learning and Reflection in the Schools

Participation in continuous, in-depth, school-based professional development must be considered part of a teacher's job (Stigler & Hiebert, 1999). Consequently, there must be systematic and systemic school district support for this critical professional activity. Indeed, in many countries whose students outperform U.S. students in mathematics, teachers spend between 15 and 20 hours per week with students and the remaining time working with colleagues on joint planning and curriculum development, pursuing classroom-relevant research, participating in ongoing teacher-led study groups, offering demonstration lessons to one another, as well as working with students and parents individually. "Thus, teaching is both more thoughtfully guided and more consciously adapted to students' learning needs in many other countries whose students achieve at high levels" (Hammond & Ball, 1997).

In fact, opportunities for reflection and analysis are central to learning to teach well. Teachers need time and encouragement to reflect on ways to improve their teaching (Stigler & Hiebert, 1999). They must see that their own learning—about mathematics and about their pupils' mathematical thinking—is critical to their instructional practice; they must learn how to inquire systematically into their pedagogical practice.

UC Ohio Survey Highlight. Overall, respondents felt that the most important changes that should be made in the delivery of mathematics education were to provide teachers with professional training to update their skills (83%), provide up-to-date computers and other technology in mathematics classrooms (77%), and to hire more teachers with strong backgrounds in mathematics (67%). Fewer than half (44%) felt that it was very important to provide teachers with more planning time.

Unfortunately, the "planning time" finding shows that there is not much public appreciation for the professional, research recommended idea of supporting teachers in their serious study of classroom learning and instruction. However, it is likely that the public does not really understand what is involved here. Also, use of the term "planning time" does not convey well what teachers should be doing during this time.

NCREL Highlight: Results from a conference with National Board Certified Ohio teachers (Otto, van der Ploeg, & Blakeslee, 2000). Teachers believe that for teacher education:

- preservice education should focus on rigorous disciplinary courses, ranges of strategies, student diversity, practical applications, connections to mentoring and staff development programs;
- professional development opportunities should be engaged in each day, should provide access to mentors, immersion in learning experiences, and encourage National Board certification;
- schools should provide in-school support for professional interaction with colleagues, flexible scheduling, support of administrators, business, industry.

Appropriate Job Support for Teachers

Staffing mathematics classes with appropriately qualified teachers requires not only strong recruitment of teachers into the field, it also demands that teachers be appropriately supported in their jobs. Important factors here include appropriate salaries, proper administrative support with discipline problems, sufficient instructional supplies, and societal respect. Certainly the modern trend of blaming teachers for educational ills without considering all the other systemic factors and policies that affect students' learning contributes to teachers' lack of job satisfaction.

Properly Aligned Assessment, Curriculum, and Teaching

The Need for High Quality Mathematics Learning Standards

To produce effective student mathematics learning, an educational system must tightly align standards for curriculum, instruction, and assessment with a comprehensive set of mathematics learning standards. These learning standards should lay out what students should know (facts, concepts, understandings, and principles), be able to do (procedures, ways of reasoning, ways of justifying, types of problems to solve), and the dispositions students should have toward the content. (Curriculum and instruction standards should specify the types of instructional experiences students should have.)

Just as mathematics instruction must be consistent with scientific research on how students learn mathematics, *it is absolutely critical that learning standards be based on the findings of modern research on students' mathematics learning*. Research can indicate the precise nature of learning goals that are achievable at each grade level, how instruction can be structured to help students achieve goals, and how different factors affect goal attainment. Setting learning standards without carefully considering research on mathematics learning dooms an educational system to inefficiency and failure.

> UC Ohio Survey Highlight. Almost all respondents (93%) indicated that they supported standards for teaching and learning mathematics and science.

The Need for High Quality Learning Standards-Based Assessment

A critical factor in keeping curriculum and instruction properly aligned with learning goals is for assessments to be completely interconnected with learning standards. Assessments must specify, measure, and report whether students meet ***specific*** content learning standards, not simply give an overall score or pass/fail judgement. Mathematics assessment should specify whether students have met the learning standards for major mathematical topics[3] and major mathematical intellectual processes[4]. Learning standards-based assessment must also assess levels of students' understanding of mathematical topics as specified by cognition-based scientific research on learning (e.g., Bright & Joyner, 1998).

Norm-referenced tests—the traditional form of assessment that merely *compares* student performance—are inappropriate for assessing students' attainment of learning standards. Instead, standards-based assessment must specify the levels of performance that must be attained on specific blocks of items to demonstrate that a given learning standard has been attained.

Also, appropriate standards-based assessment cannot be restricted to multiple choice items in which students merely choose answers from a list (Glaser & Silver, 1994). To assess the broad range of learning goals specified by NCTM and suggested by research, assessment must include items in which students formulate, provide, and explain their answers. "Given the current interest in promoting complex reasoning and problem solving, the fact that these assessments have tended not to include questions in which students are required to produce their own answers, to display the processes used to obtain an answer, to explain the thinking or reasoning associated with their response, or to exhibit alternative approaches to or interpretations of a problematic situation has severely limited the extent to which they are seen as related to important curricular goals" (Glaser & Silver, 1994, p. 15). As the National Research Council (1989) states, "We must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics" (Glaser & Silver, 1994, p.

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[3] Examples of major mathematical topics are whole numbers; whole number operations; fractions; fraction operations; properties of 2D figures; properties of 3D figures; length, area, and volume measurement; solving linear equations in one variable, graphing quadratic equations in two variables.

[4] Examples of major mathematical intellectual processes are problem solving, justification and proof, communication, use of symbolism and representations, mathematical modeling.

70).

> <u>UC Ohio Survey Highlight</u>. Only 49% of respondents felt that the Ohio proficiency tests were a fair measure of how well students learn mathematics.

Cognition Based Assessment

High-quality and valid assessment must be cognition-based. That is, it must be firmly linked to scientific research on students' mathematics learning, something that is sorely missing in traditional assessment paradigms (Battista, 1999a; Lesh, Lamon, Behr, & Lester, 1992; Masters & Mislevy, 1993; Webb, 1998). To be consistent with such research, *assessment must be situated within research-based descriptions of students' mathematical cognitions* (Goldin, 2000; Lesh & Kelly, 2000); "assessment that is no longer limited to discrete, low-level mathematical skills requires…a sound cognitive model for describing the capabilities to be assessed" (Goldin, 1992, pp. 67-68).

A cognition-based assessment system (CBAS) should include the following (Battista, 2001):

- Descriptions of the core principled, generative mathematical ideas and reasoning processes around which students' sense making and understanding of mathematics cohere.
- For each core idea, and within appropriate cognition-based theoretical frameworks, descriptions of the cognitive processes, milestones, and fluencies that underlie and compose students' development of the idea throughout school.
- For each core idea, coherent sets of assessment items that enable users to characterize students' cognitions and precisely locate students' positions in the "constructive itineraries" typically taken in acquiring competence with the idea.

A CBAS can (a) support teachers who are trying to implement the type of cognition-based reform instruction recommended by research on students' mathematics learning and professional organizations; (b) motivate teachers who retain traditional teaching techniques to change their instruction when the CBAS reveals that their students are not learning mathematics as well as they assume; (c) provide a teacher-education vehicle to conceptually deepen teachers' understanding of core ideas in K-5 mathematics and enlighten teachers about how students learn those ideas; and (d) provide educators and researchers interested in examining the effectiveness of various curricula and instructional methods with an assessment system that can be used to evaluate the genuine progress students are making in acquiring powerful and deep understanding of core mathematical ideas (Battista, 2001).

Assessment and Reform

Assessment plays a critical role in this reform in two fundamental ways. First, appropriate assessment of student learning is a critical component in implementing instruction and curricula consistent with modern research on students' learning (Battista, in press; Black & Wiliam, 1998). Second, to validly evaluate the effectiveness of various education reforms and mathematics programs, assessments that precisely describe the *quality* of students' mathematics learning are critical (NSF-IMD RFP).

And there are two fundamentally different approaches to assessing student learning (Battista, 2001). Most instructional effectiveness comparisons, accountability assessments, and teacher-constructed tests measure student learning with testing instruments that determine <u>if</u> students have acquired particular mathematical knowledge, skills, types of reasoning, and so on (often simply by comparing student performance). In contrast, techniques developed in modern research on students' mathematics learning enable researchers to investigate more precisely <u>what</u> students learn, along with the factors (both mental and social) that affect that learning. Such research utilizes qualitative microgenetic studies that carefully examine the exact nature of students' cognition. Unfortunately, assessment of the "what" type is almost non-existent in non-qualitative research settings, in classroom practice, in high-stakes testing, and most curriculum comparison studies.

To truly understand the effects of instruction—be it reform versus traditional teaching, or the differential achievement of various groups of students—and to properly design and guide instruction, *mathematics educators need both the "if" and the "what" approaches to assessment.* On the one hand, after learning goals have been set, we need to determine if students achieve these goals. On the other hand, we must determine more precisely what ideas and reasoning students develop as they participate in mathematics instruction.

<u>Public and Parental Support of Education</u>

Although most people are strongly concerned about the quality of public education, only about half have taken any action to help schools (Fredreka Schouten, 4/17/01, *USA Today*). In fact, an *Education Week* survey found that 55% of survey respondents said they would be motivated to get involved in education only by a crisis such as a school shooting. Only 37% said low test scores would inspire their involvement (Schouten, 4/17/01). And only 22% of respondents said that people in their community take a lot of responsibility for ensuring quality in public schools.

Parental Involvement in Their Children's Schools and School Work

Given the public apathy reported above, it is no surprise that increasing parent involvement in their children's education has been touted by many as an important factor in improving students' academic success. This notion is bolstered by international comparisons of mathematics education. For instance, Stevenson et al. (1986) investigated the reasons for the high academic achievement (AA) of Chinese and Japanese children compared with US children. Their studies indicated greater attention to academic activities and success among Chinese and Japanese children and parents. "Members of the 3 cultures differed significantly in terms of parents' interest and involvement in their children's AA, parents' beliefs and expectations concerning their children's AA, and parents' and children's beliefs about the relative influence of effort and ability on AA" (Stevenson et al., 1986).

However, although some research suggests a positive relationship between parental involvement and student academic success (Shaver & Walls, 1998), caution must be exercised in interpreting the results. First, of the research that has shown a positive relationship between parental involvement and student achievement, most has been correlational in nature. Such research cannot establish a causal relationship between parental involvement and student academic success. That is, the fact that parental involvement is positively related to student academic success does not imply that increased parental involvement causes increased achievement. It could be, for instance, that parents who are more involved in schools become involved because of the higher value they place on schooling, which they transmit to their children, which in turn, makes their children more motivated to do well in school. So the causal factor is not involvement in schools, but a value system that is consistent with and supports schooling.

Second, in examining effects of parental involvement, researchers have not adequately distinguished between different types of parental involvement. For instance, Sui-Chu & Willms (1996) found that parental involvement as school volunteers and with PTO meetings differed significantly among schools, but had only a modest effect on students' reading achievement and a negligible effect on their mathematics achievement. In contrast, discussing school activities at home and helping children plan their programs had the strongest effect on academic achievement, but did not differ significantly among the studied schools. So the aspects of parental involvement that varied among schools did not affect achievement much; whereas the aspects of involvement that did not vary affected achievement. Other studies too have looked at parental involvement more carefully. For instance, even when parents are positively involved in their children's learning, they sometimes confuse or frustrate their children while trying to help them (Balli, 1998).

Looking deeper into this issue, research suggests that (a) American students have low motivation for school work, and (b) student motivation is strongly affected by the beliefs about schooling held by parents and society and passed on to children. On the first point, researchers have found lower motivation for academic learning among American students than students in China and Japan, two countries whose students consistently outperform American students in international studies of mathematics achievement (Stevenson, 1995). Indeed, time spent studying outside of school and percent of time attending to lessons in school are both lower for American than for Chinese and Japanese students. Furthermore, American students report that they spend more time on jobs, dating, and socializing with friends than do Chinese and Japanese students.

On the second point, one belief that seems to affect student motivation to expend time and effort on academic activities is satisfaction with academic success. Parents and students who are satisfied with students' school performance apparently see little reason for students to take school more seriously or study harder (Stevenson, 1995). Consistent with this view, Chinese and Japanese parents and their children are less satisfied with the children's school performance, while American students and their parents exhibit excessive confidence in students' academic abilities and performance. "American mothers did not require their child to demonstrate high levels of academic achievement for them to be satisfied. Nor were they dissatisfied unless the child's performance was notably below average….From the perspective of these American mothers, it is better for children to be bright than to be good students. The situation is very different in Taiwan and Japan. Going to school and doing well academically are the children's two major responsibilities" (Stevenson & Lee, 1990, p. 97). And perhaps one reason for the lack of dissatisfaction among American parents and students is the lack of clear national learning standards for mathematics (unlike mathematics education in China and Japan). Thus, according to Stevenson & Lee, "Two factors that work strongly against high achievement by American children are the low academic standards held by parents and the overestimations that parents make of their children's abilities" (p. 100).

Another factor that affects students' academic motivation is their beliefs about factors that influence school achievement. As an explanation for achievement, American parents and students place greater emphasis on ability; Chinese and Japanese parents and students place greater emphasis on effort (Stevenson & Lee, 1990, p. 97). In fact, Stevenson & Lee argue that the belief that success in school depends on ability rather than effort makes effortful participation in academic activities less likely. Parents who believe in the primacy of ability may question such activities for low ability children because they believe that these students will fail no matter how much effort they expend, and for high ability children because they believe that these children do not need to exert much effort to succeed. "A greater emphasis on ability appears to be related, therefore, to American children's lower accomplishments" (Stevenson & Lee, 1990, p. 66). Indeed, in China, "children accept the philosophy that the major path to success is through effort, and they incorporate their parent's beliefs about the importance of academic achievement. They enter school with a clear purpose and a willingness to work hard. Because they have internalized the value of learning, their motivation remains high throughout the elementary school years" (Stevenson & Lee, 1990, p. 99). (Keep in mind, however, that student motivation also depends heavily on the quality of classroom instruction, which is discussed in other parts of this paper.)

In conclusion, while it seems worthwhile to encourage parents to get involved in their children's schooling, care and thought must be given to the nature of that involvement. First, if parents are encouraged to help students with homework, parents must be aided and guided with this task. Schools must help parents genuinely understand the goals of the school curricula and homework. Schools must also help parents learn how to help their children in ways that support rather than impede school curriculum goals. However, for most parents to genuinely aid their children with school work in modern mathematics curricula, parents themselves would need instruction on mathematics and pedagogy—something that most schools do not have the resources to provide.

Second, and by far most important, parents must be encouraged to show strong interest in their children's academic progress and to demonstrate, as well as strongly encourage, a high value for academic achievement. Parents must emphasize and strongly encourage hard work in school, promoting in their children the belief that school learning is critical and that effort pays off in academics (and life).

To accomplish this change in values among parents and children requires a change in the values and belief structures of the whole American culture. Academics must be more strongly valued throughout our society. Business, education, and political leaders need to make a concerted effort in word and deed to change these values. For instance, they could cooperatively sponsor TV campaigns that help adults and children understand the importance of mathematics education, school learning, and hard work in schools. But the effort cannot stop with publicity. Businesses and universities must require students to demonstrate academic success before admitting them—this is the "bottom line" for most students. As long as businesses and universities do not require high standards of academic success in hiring and admittance, the value of education in our culture will remain diluted.

SECTION 3: PROFILE OF THE CURRENT CONDITION OF MATHEMATICS EDUCATION IN OHIO

Profile of Ohio Mathematics Education

Ohio's current mathematics education system is typical of that in the United States. It focuses on rote skills rather than concepts, uses curricula that are a "mile wide and an inch deep," favors teacher-centered rather than student-centered instruction, utilizes inappropriate textbooks, and exhibits student achievement below international averages and below that of many of our economic peers (Schmidt, McKnight, & Raizen, 1997).

Traditional instruction focuses on skills

According to NCREL researchers (van der Ploeg, 2001), when asked for the main thing they wanted students to learn from a lesson, 61% of U.S. eighth-grade mathematics teachers responded “skills,” while only 21% answered “thinking.” In contrast, responses from Japanese teachers, whose students score near the top in most international mathematics achievement comparisons, were reversed; 73% said “thinking,” while 25% said “skills.” Consistent with this finding, 72% of U.S. fourth-grade and 59% of eighth-grade mathematics teachers report they have their students practice computational skills during most or all lessons. In Ohio, the comparable percentages for practicing computational skills are higher: 82 percent in third and fourth grade and 74 percent in seventh and eighth grade. Even in Ohio’s 12th-grade classes, computational practice occurs in most or all classes, according to 58 percent of the mathematics teachers (van der Ploeg, 2001). In addition, typically, in Ohio and the rest of the U. S., homework consists of repetitive practice exercises. As a group, U.S. teachers spend more time reviewing homework in class than teachers in other countries. Teachers in most other countries spend little or no time on homework in class.

Traditional instruction is teacher-centered

Although research suggests that learning is enhanced when students do most of the work during a mathematics lesson rather than watching the teacher do the work for them, U.S. and Ohio schools continue to be teacher-centered. Indeed, in Japan, students are asked to perform 40 percent of the mathematics work in class, whereas U.S. students are asked to do only about 9 percent of the work (van der Ploeg, 2001). Furthermore, although cooperative group instruction can produce higher student achievement if properly implemented, such instruction is not used often in U.S. nor Ohio schools (approximately 10% of teachers used it consistently) (van der Ploeg, 2001).

> <u>NCREL Highlight:</u> TIMSS "results revealed great differences between American classroom practice and most especially Japan's, where student achievement in mathematics…is consistently higher. Overall, Japanese teachers use methods that, according to current research, are proven to be effective. By contrast, U.S. teachers use more outdated practices and techniques" (Otto & van der Ploeg, 2001c, p. 1).

Breadth rather than depth in Ohio mathematics curricula

Learning tasks chosen by Ohio's teachers are not unlike those in the rest of the nation's classrooms (van der Ploeg, 2001). About 20% of Ohio mathematics teachers report that they explain the reasoning behind ideas in every mathematics lesson they teach; 50% claim they explain the reasoning most of the time; and 30% only some of the time. Twelfth-grade teachers are the most likely to explain ideas most or all of the time. Ohio mathematics teachers are much more likely to ask older students to work on problems for which there is no obvious solution, a practice that is equally important for students of all ages.

Mathematical Content: Breadth rather than Depth

In examining patterns of mathematics topics to be taught, researchers concluded that the Japanese pattern is consistent with a system that establishes the basics thoroughly before middle school and thereafter focuses the curriculum each year on a small number of new topics. The American and Ohio patterns appear more consistent with systems that introduce many topics early but none deeply and thereafter repeat the topics annually, presumably deepening instruction each time. NCREL researchers also found that Ohio school districts' mathematics curricula are not challenging or conceptually deep, that Ohio students are not expected to learn much in the elementary grades, and that topics are repeated over and over in Ohio classrooms (van der Ploeg, 2001).

Indeed, most Ohio teachers spend less than 5 lessons on any one mathematics topic, merely scratching the conceptual surface of the topics. No doubt that is why so many topics have to be retaught so frequently; 30 to 40 percent of Ohio teachers report spending time on whole numbers, fractions, and percents, with more than half spending at least a week on each (van der Ploeg, 2001). In sum, the data suggest that Ohio teachers focus more on breadth rather than engaging students in a deep, meaningful, and satisfying study of mathematics (van der Ploeg, 2001).

Textbooks, Especially Poor Ones, Dominate Ohio Teaching

According to NCREL researchers, local district and school curriculum guides dominate the resources Ohio teachers use to decide what to teach (van der Ploeg, 2001). Ohio Proficiency Test guidelines play a major role in grades where the tests are given. Ohio teachers rarely cite as resources the Ohio model mathematics curriculum or NCTM Standards. Instead, textbooks dominate Ohio teachers' decisions about how to teach mathematics. Even more, the large variety of mathematics textbooks in use and the limited teaching of algebra suggest that, despite the relative clarity of Ohio's model mathematics curriculum and the relative specificity of the Proficiency Test guidelines, there is little consensus among districts, schools, and teachers about what should be taught and what constitutes a good textbook (van der Ploeg, 2001).

Moreover, Ohio mathematics teaching is dominated by inappropriate textbooks. Indeed, the two middle school textbooks most often used in Ohio are both rated "unsatisfactory" by AAAS' Project 2061, while texts rated "satisfactory" or "acceptable" are used by only 7% of Ohio teachers (van der Ploeg, 2001). Only about 16 percent of Ohio's seventh and eighth grade mathematics teachers use a textbook rated exemplary or promising by a U.S. Department of Education expert panel (van der Ploeg, 2001).

Unmotivated students

Many Ohio students see few incentives to do well in school or school mathematics (van der Ploeg, 2001).

> NCREL Highlight: Recent studies of American high schools suggest that many students see little reason to take school seriously. Over 50 percent of students say they could bring home grades of C or worse without their parents getting upset. Twenty-five percent say they could bring home grades of D or worse without upsetting parents. Nearly 20 percent of students "do not try as hard as they can in school because they are worried about what their friends might think." Over one-third get through the school day primarily by "goofing off with friends." (Otto & van der Ploeg, 2001d, p. 1)

> NCREL Highlight: "Companies that hire high school graduates rarely, if ever, ask to see transcripts or school records." Thus, there need to be more incentives for students to take schooling more seriously. One incentive is to make school more interesting and relevant for students. Another is for universities and businesses to require higher educational standards for entrance. (Otto & van der Ploeg, 2001d, p. 1)

Recent State-Wide Initiatives

Recent attempts to improve mathematics instruction in Ohio include use of Proficiency Tests and District Report Cards; writing of the Ohio Model Curricula; professional development programs such as National Science Foundation/Ohio sponsored systemic initiatives; development of a new Teacher Licensure system; continuation of Projects *Discovery* and SUSTAIN. Another force for improving mathematics education in Ohio is the Ohio Mathematics and Science Coalition.

Problems with Traditional Mathematics Education

As we have seen, mathematics education in Ohio is traditional; it is very much like the mathematics education that is being implemented and has been implemented for decades across America. This section describes the major problems that exist in traditional American mathematics education.

Traditional Mathematics Teaching: The Wrong Mathematics Taught in the Wrong Way

Much of the failure in school mathematics is due to a tradition of teaching that is inappropriate to the way most students learn (NRC, 1989, p. 6).

Mathematics teaching in the United States has changed little over the last 50 years. Despite the fact that numerous scientific studies have shown that traditional methods for teaching mathematics are not only ineffective but seriously stunt the growth of students' mathematical reasoning and problem-solving skills (Battista & Larson 1994; Lindquist 1989), and despite professional recommendations for fundamental changes in mathematics curricula and teaching (NCTM, 1989, 2000; AAAS, 1999), traditional teaching continues almost unabated (Hiebert 1999, NCES, 1996).

In traditional mathematics instruction, teachers demonstrate, while students memorize and imitate, the use of formal symbolic procedures. Despite research indicating that learning that emphasizes sense making and understanding, rather than memorization, produces better facility in "transferring" that learning to new situations (Bransford, Brown, & Cocking, 1999; Mayer & Wittrock, 1996), traditional instruction places more emphasis on memorization and imitation than on understanding, thinking, and reasoning (De Corte, Greer, & Verschaffel, 1996; Greeno, Collins, & Resnick, 1996; Silver & Stein, 1996). Indeed, according to TIMSS, for close to 80% of the topics covered in 8^{th} grade American mathematics classes, teachers merely demonstrated or stated procedures, not explained or developed them, and 90% of students' seatwork involved practicing procedures that had already been demonstrated (Hiebert, 1999). Furthermore, even when traditional instruction attempts to promote understanding, students fail to make personal sense of the ideas because classroom derivations and justifications are too formal and abstract (Battista & Larson, 1994; Bransford, Brown, & Cocking, 1999). Thus, for most students, school mathematics is an endless sequence of memorizing and forgetting facts and procedures that make little sense. The National Research Council dubbed the "learning" produced by such instruction as "mindless mimicry mathematics." Instead of understanding what they are doing, students parrot what they have seen and heard.

Traditional Mathematics Learning as Academic Rituals

Because traditional instruction focuses so much on symbolic computation procedures, many students come to believe that mathematics is mainly a matter of following formal rules, that it consists mostly of symbol manipulation, and that the rules and symbols have no connection to their own intuitive ideas about dealing with realistic situations (De Corte, Greer, & Verschaffel, 1996; Hiebert & Carpenter, 1992). In fact, students in traditional classrooms believe that the school "mathematical" tasks they engage in have no "real-world" value, but rather are simply what they have to *do* to meet their obligations in the social situation of the classroom (Nickson, 1992).

Instead of seeing mathematics as a worthwhile intellectual endeavor involving exploration, reflection, and discussion, students see it as a set of procedures to be learned through imitation. Doing mathematics is an academic ritual that has no genuine usefulness. Such ritualistic mathematics, stripped of its power to explain anything that matters, and devoid of the interconnections that arise from sense making, becomes a hodgepodge of memorized—and easily forgotten—rules.

In sum, traditional instruction unintentionally encourages students to believe that doing mathematics consists of following procedures that have little or no relation to personal sense making or needs. This belief, in turn, causes students to focus on imitation rather than conceptual understanding, leading to disastrous results.

> UC Ohio Survey Highlight: Ninety-five percent of Ohioans agreed with the statement "Math & science should help students make sense of the world around them;" 80% agreed a lot with the statement. Thus, mindless mimicry mathematics is not what Ohioans want from their mathematics education system.

Disconnections between Student Real-Life Sense Making and School-Taught Mathematical Symbol Manipulation

An example from Paul Cobb of Vanderbilt University poignantly illustrates the disconnect between students' mathematics and school mathematics. Second-grader Auburn was asked to solve the horizontal sentences 16+9=__, 28+13=__, 37+24=__, and 39+53=__. For each problem, she counted-on by ones as she sequentially put up fingers. (So , for example, for 16+9, she counted 16—17, 18, 19, 20, 21, 22, 23, 24, 25.) Two days later, Auburn was asked to complete a worksheet that presented these same problems in standard vertical form.

16	28	37	39
+ 9	+13	+24	+53

For each of these tasks, Auburn first added the ones column then the tens. However, she failed to "carry" and produced answers of 15, 31, 51, and 82. As soon as she completed the worksheet, the interviewer presented the horizontal sentence 16+9=___ and Auburn counted-on and gave 25 as her answer.

I: So when we count we get 25 and when we do it this way (points to worksheet) we get 15. Is that OK to get two answers or do you think there should be only one?
A: (Shrugs shoulders.)
I: Which one do you think is the best answer?
A: Twenty-five.
I: Why?
A: I don't know.
I: If we had 16 cookies and 9 more, would we have 15 altogether?
A: No.
I: Why not?
A: Because if you counted them up together, you would get 25.
I: But is this (points to answer of 15 on the worksheet) right sometimes or is it always wrong?

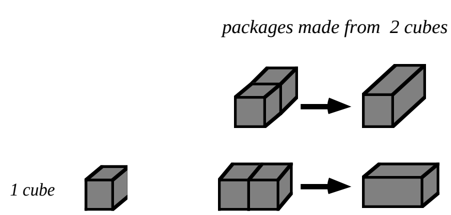
A: It's always right. (Cobb, 1988, p. 98)

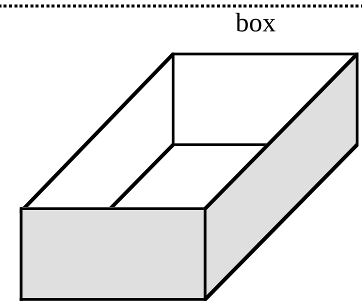
As Cobb comments “The contrast between Auburn's behavior in the two parts of the interview and her final comment indicate that the arithmetic she was currently studying in school had nothing to do with the world of physical objects and real-life problems or her self-generated methods. For her, school arithmetic seemed to be an isolated, self-contained context in which the possibility of doing anything other than attempting to recall prescribed methods did not arise” (1988, p. 98). Auburn's loss of intellectual autonomy in the worksheet setting was almost complete. Authority had indicated that this was the correct method; it did not matter that the answer made no sense to her or that it was inconsistent with her self-generated methods for thinking about this type of situation.

Even Our Brightest Students' Learning Is Inadequate

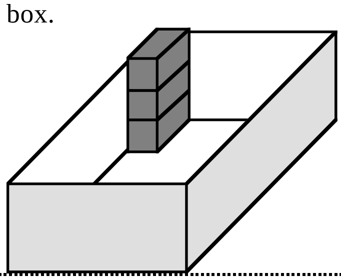
The effects of the instructional focus on imitating symbolic procedures rather than personal sense making, are not restricted to "lower level" students; even the brightest students are being affected. For instance, a bright Ohio eighth grader in a highly regarded Ohio school system who was three weeks from completing a standard course in high school geometry—so she was two years ahead of schedule for college prep students—responded as follows on the problem below (Battista, 1998).

Collin has some packages that each contain two identical cubes. He wants to know how many of these **packages** it takes to completely fill the rectangular box below.

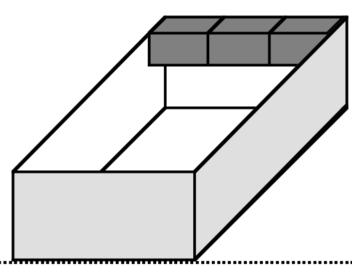




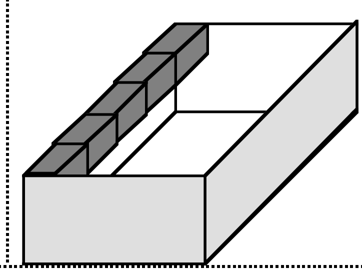
Collin knows that he can fit 3 packages along the height of the box.



He knows that he can fit 3 packages along the width of the box.



He knows that he can fit 5 packages along the length of the box.



Student: *It's 45 packages[5]. And the way I found it is I multiplied how many packages could fit in the height by the number in the width, which is 3 times 3 equals 9. Then I took that and multiplied it by the length, which is 5, and came up with 9 times 5, which is 45.*

Obs: *How do you know that is the right answer?*

Student: *Because the equation for the volume of a box is length times width times height.*

Obs: *Do you know why that equation works?*

Student: *Because you are covering all three dimensions, I think. I'm not really sure. I just know the equation.*

This student did not understand that the mathematical formula she applied assumed a particularly structured mathematical model of a real-world situation, one that was inappropriate for the problem at hand. Indeed, research shows that despite its simplicity, genuinely understanding why the volume formula ($V=l\times w\times h$) works is surprisingly difficult for students (Battista & Clements, 1996). Although this bright student had learned an impressive amount of routine mathematical procedures, the above example illustrates that much of the learning she accomplished in her accelerated mathematics program was only at the surface level, a finding that is all too common among bright students. Indeed, only 38% of the students in her geometry class answered the item correctly, despite the fact that all of them had scored at or above the 95^{th} percentile on a widely used standardized mathematics test in 5^{th} grade, all had already passed the state's 9^{th} grade proficiency test in mathematics, and the class was taught by an excellent teacher. Because such students have the capability to make sense of mathematics if given the chance, the case could be made that these students, more than any others, are being terribly shortchanged by traditional mathematics instruction.

The American Public's Lack of Understanding of Mathematics, How Students Learn Mathematics, and Educational Deficiencies

A major impediment to improving students' mathematics learning is adults' lack of knowledge—both of mathematics and of research on how students learn mathematics. Because mathematics has been taught so poorly for so long, few adults have a genuine understanding of mathematics or the mathematical enterprise. Most adults, who themselves have been mathematically miseducated, believe that mathematics involves, not sense making, but following rigid sets of rules invented by others. They have learned, and expect others to learn, mathematics in this way.

The situation is worse when it comes to scientific knowledge about students’ mathematics learning. Not only do most adults not understand the scientific research on mathematics learning, too often the educational programs and methods used in schools are formulated by practitioners, administrators, laypersons, politicians, and professors of education, with a total disregard for this research.

[5] Actually, 90 packages fit in the box.

Public Misconception of Educational Deficiencies

In general, the public exhibits little understanding of deficiencies in American mathematics education. Hershberg suggests that the public naively believes that all we have to do to close the skills gap between American students and students in our economic competitor nations is to "kick the lazy students, teachers and school administrators in the pants" (2000, p. 34). However, he argues that unless American schools change their pedagogy to focus on developing the problem-solving skills students need for success in the new economy, the school reform-standards-accountability movement will fail, and the gap between the “haves and have-nots” will grow larger, threatening the middle-class center that is the foundation of our nation.

Hershberg further argues that the majority of Americans, those who live outside large cities, mistakenly believe that the schools their children attend are doing a fine job (2000). They are complacent about their local schools' quality and think that all the problems with American education that they hear and read about occur elsewhere and do not affect them directly.

The Mathematics Education System Perpetuates Itself

Teachers most often teach the way they were taught. Because almost all current teachers were educated at the elementary, secondary, and university levels in curricula that promote the conception of mathematics as imitation rather than sense making, their understanding of and beliefs about mathematics make it very difficult for them to use new and improved pedagogical methods and curricula (Battista, 1994). Thus, the traditional mathematics curriculum perpetuates itself; we are caught in a pernicious cycle of mathematical miseducation.

We might think that one place to break this cycle of miseducation is with mathematics courses taught in teacher education programs. Unfortunately, most university mathematics courses reinforce rather than debunk the view of mathematics as a set of procedures to be memorized. As described by the National Research Council, they present mathematics “only in the authoritarian framework of Moses coming down from Mt. Sinai.” (It is also true that teacher-inservice programs focus so much on general education courses that teachers almost never take a sufficient number of courses dedicated to mathematics and teaching mathematics.)

Indeed, all of undergraduate mathematics education is in need of reform. Because we now live in a highly competitive and technological <u>worldwide</u> economy, America is struggling to move from a mathematics education system for educating the elite few who are to become mathematicians and scientists to educating the broad workforce. "Given…21st century economies, it is not good enough that we produce a sufficient elite corps of science, math, and engineering professionals. We must raise levels of math, science, and technology literacy throughout our society" (USCNS, 2001). The traditional "filtering" approach utilized in university mathematics programs is insufficient for current individual and societal needs.

Consequently, according to Hyman Bass, esteemed mathematician and current president of the American Mathematical Society (AMS), academic mathematical scientists, who typically spend at least half of their professional lives teaching, must reconsider their role as educators, paying more and deeper attention to pedagogy (1999). Unfortunately, the disposition of many mathematicians toward the problems of education "implicitly demeans the importance and substance of pedagogy" (Bass, 1999). Most mathematical scientists typically address educational issues exclusively in terms of subject matter content and technical skills, ignoring the issue of the cognition that underlies learning. Furthermore, "many mathematical scientists have tended to look upon education professionals with doubts bordering on ill-disguised contempt" (Bass, 1999). Echoing this sentiment, the Conference Board of Mathematical Sciences stated that "the reality today is that…there is considerable distrust between mathematicians and mathematics educators" (2000).

To remedy this problem, serious efforts are needed to foster cooperation, mutual understanding, and respect between mathematicians and mathematics educators. Critical to enhancing America's undergraduate mathematics education system is the need to "establish contexts for respectful communication and professional collaboration between mathematical scientists and education professionals" (Bass, 1999).

> UC Ohio Survey Highlight. Only 50% of respondents believed that colleges and universities are doing a good job of preparing mathematics teachers.

Flawed Assessment

Most school districts rely heavily on standardized and state “proficiency” tests as “bottom line” measures of their students’ learning progress, despite the fact that such assessments inadequately measure increases in students' understanding and reasoning in reform mathematics curricula and overestimate the quality of mathematics learning in traditional curricula (Battista, 1999c; Cooper, 1998; Lamon et al., 1996; Romberg et al., 1992). This practice has several undesirable consequences. First, because most high-stakes tests measure traditional rather than reform outcomes, their use maintains the inertia of traditional instruction and seriously impedes adoption of reform practices. Second, such tests rarely are consistent with scientific research on what mathematical understandings might reasonably be expected of students at various grade levels. Consequently, teachers, guided by the demands for "coverage" of content and pressured by administrators and parents to ensure that students pass such high-stakes tests, often demand that students use abstract mathematical procedures before students can make personal sense out of them. Students are thus forced either to “drop out” of mathematics learning or to resort to mindless mimicry.

Test Preparation Versus Teaching

Because one of the consequences of testing is to signal to students, teachers, and the public those aspects of learning that are valued, when assessments are used for state and district accountability purposes, teachers are pressured by the public and school administrators to spend increasing amounts of time preparing for the tests (Romberg et al., 1992). Poor understanding of the testing process creates the "teach to the test" phenomenon observed in so many school systems. Overly mindful of state-mandated proficiency tests, instead of teaching mathematical concepts and reasoning, most school programs rotely teach students how to solve the specific types of problems that appear on these tests. Test preparation replaces huge amounts of genuine instruction, with students spending most of their time working repetitively on questions that mimic assessment items, reducing the curriculum to mimicry mathematics. Teaching degenerates into a narrow type of instruction that focuses on practice and memorization of decontextualized fragments of mathematics instead of promoting understanding and problem-solving ability.

There is an abundant amount of evidence that such externally mandated assessments can limit and negatively affect the quality of mathematics instruction (Glaser & Silver, 1994). In particular, research suggests that teachers tend to narrow their instruction by giving a disproportionate amount of their time and attention to teaching the low-level specific content most heavily assessed rather than teaching underlying concepts or overarching principles.

Accountability Uses of Tests Give Inflated Assessments of Learning

There is also a great deal of evidence that "focused teaching to the test encouraged by accountability uses of results produces inflated notions of achievement" (Linn, 1998, p. 10). The evidence suggests that gains on initial administration of high-stakes standardized tests are due more to "learning to take the test" rather than increases in genuine mathematical knowledge. These findings, together with evidence that high-stakes use of standardized tests leads to a narrowing of the curriculum and an overemphasis on basic skills, strongly suggest that changes must be made in the nature of assessments and the degree to which they are aligned with the types of learning specified in content standards (Linn, 1998).

Testing Can Exacerbate Equity Issues

High-stakes tests can exacerbate equity issues. For instance, one study reported that 74% of the teachers of high-minority classes began test preparation activities at least 1 month before an externally mandated assessment, and more than 30% spent at least 20 hours on test preparation. However, only 32% of teachers of low-minority classes spent 1 month or more on test preparation before the assessment, and only 9% reported spending 20 or more hours (Glaser & Silver, 1994). "The problems appear to be worst in urban school districts, where more than 60% of mathematics and science teachers report that externally mandated assessments have a negative impact on their curriculum or instruction" (Glaser & Silver, 1994, p. 17).

Assessments Do Not Support Teachers in Their Instructional Efforts

Finally, a National Institute of Education report of a conference on assessment and instruction noted: "Current testing procedures are not helpful to teachers or students in their day-to-day efforts to teach and learn," and "present day testing programs are largely extraneous to everyday classroom teaching" (Glaser & Silver, 1994, p. 15). There is a need for test formats to be more aligned with learning goals, and test results to be more useful for instructional decision making (Glaser & Silver, 1994).

Mathematics Textbook Industry

Because commercial textbook companies are for-profit organizations, for the most part, they publish what will sell, regardless of scientific research on students' mathematics learning and professional recommendations. Because most teachers do not have a firm understanding of research-based teaching but are familiar with highlights of the professional recommendations, they are easily sold on commercially available mathematics textbooks that consist of traditional curricula with enough superficial changes tacked on so that publishing companies can market them as "new" and consistent with professional recommendations. For the most part, textbook companies produce mathematical curricula that are mere caricatures of professionally recommended, scientifically sound instruction. Such textbooks are a powerful factor in maintaining the stranglehold of traditional mathematics instruction.

Myths and Folklore

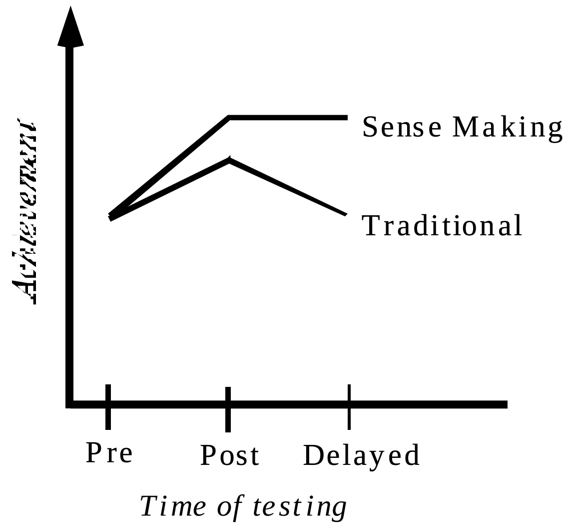
The methods of traditional mathematics teaching are based, for the most part, on folk and outdated psychological theories. Typical of such theories are their inclusion of myths.

"Covering" Material: Teaching without Learning

One of the major components of traditional mathematics teaching is the almost universal belief among mathematics teachers in the "myth of coverage." According to this myth, "If mathematics is 'covered,' students will learn it." This myth is so deeply imbedded in traditional mathematics instruction that, at each grade level, teachers feel tremendous pressure to cover huge amounts of material at breakneck speeds. It has encouraged a curriculum that is overly broad and lacks conceptual depth (Schmidt, McKnight, & Raizen, 1997). It has encouraged acceleration rather than careful attention to understanding, even for the brightest students. Belief in this myth causes teachers to criticize reform curricula as inefficient because students in such curricula study fewer topics at each grade level.

Research Debunks This Myth

Research, however, has uncovered the fundamental flaw in the "coverage" myth, as shown in the figure below (Bell, 1989). Because students in traditional curricula learn ideas and procedures rotely rather than meaningfully, they quickly forget them, so the ideas must be retaught year after year. In contrast, in sense-making curricula, because students retain learned ideas for long periods of time, and because a natural part of sense making is to interrelate ideas, rather than repeatedly forgetting the same ideas, students accumulate an ever-increasing store of well-integrated knowledge.



In fact, Alan Bell of the Shell Centre for Mathematical Education at the University of Nottingham turns the myth-based reasoning of coverage on its head: "It may be felt that there is no time for a method which involves intensive discussion of particular points. But on the evidence presented...we have to ask whether we can afford to waste pupils' time on [traditional] methods which have such little long term effect when...we could be doing so much better" (1989).

Indeed, consistent with Bell's claim, TIMSS data suggest that Japanese teachers, whose students significantly outperform U.S. students in mathematics, spend much more time than U.S. teachers having students delve deeply into mathematical ideas (NCES, 1996). Bransford, Brown, & Cocking suggest why focusing on coverage does not work: "Attempts to cover too many topics too quickly may hinder learning and subsequent transfer because students (a) learn only isolated sets of facts that are not organized and connected or (b) are introduced to organizing principles that they cannot grasp because they lack enough specific knowledge to make them meaningful" (1999, p. 46).

Skill Must Come Before Understanding.

Many people, including many teachers, believe that students, especially those in lower level classes, should "master" mathematical procedures first, then later try to understand them. For instance, Wadsworth found "that 86 percent of the public believes kids should memorize the multiplication tables and learn to do mathematics by hand before they go on to use calculators and computers. Otherwise they will never fully understand mathematics concepts" (1997, p 15). "It is not that people think higher-order skills are unimportant, but rather that they are even more concerned about two other issues. The first is the belief that until the basics are acquired, nothing else can be learned at all. Second, they believe that many children in today's schools are not acquiring the basics" (Wadsworth, 1997, p. 17).

However, research indicates that if students have already memorized procedures through much practice, it is very difficult for them to later return to understand them (Hiebert, 1999; Mack, 1993; Pesek & Kirshner, 2000). For example, Wearne and Hiebert found that fifth and sixth graders who had practiced rules for adding and subtracting decimals by lining up the decimal points were less likely than fourth graders with no such experience to acquire conceptual knowledge from meaning-based instruction on decimals (Hiebert & Carpenter, 1992). In fact, researchers have found that the traditional practice of having students master computation skills then giving them relevant word problems is misguided. "Word problems should not be treated as applications of previously learned computational procedures, but as opportunities for problem solving and sense making. They should be viewed as a way to introduce new mathematical concepts" (Battista & Larson, 1994).

Teacher Beliefs and Knowledge that Are Inconsistent with Professional Recommendations and Research Cause Poor Implementations of Good Curricula

Having an even greater effect on instruction than myths are teachers' beliefs and knowledge. Research has shown that teachers' knowledge and beliefs about mathematics, mathematics learning, and students' mathematical thinking can heavily influence the ways they teach. Unfortunately, though an abundance of scientific research has shown that traditional mathematics instruction is woefully ineffective, the majority of U.S. mathematics teachers still adhere to teaching that is traditional at its core (NCES, 1996). Furthermore, even though a fair number of teachers say they are familiar with teaching methods advocated by NCTM and even report using such methods, research has shown that teacher-reports greatly overestimate how consistent teacher practice is with these methods (Spillane & Zeuli, 1999). Finally, research shows that a great many U.S. mathematics teachers are deficient in knowledge of mathematics, knowledge of how students think about and construct mathematical ideas, and skill in creating instructional classroom environments in which students can inquire and make sense of mathematics (e.g., Grouws & Cebulla, in press; Ma, 1999).

Teacher Beliefs and Knowledge Insufficient to Implement Professional and Research-Based Recommendations for Instruction

Although most U.S. commercial textbooks are still traditional at their core, there are U.S. curricula, developed with support from the National Science Foundation, that implement modern professional recommendations with high fidelity. Research conducted during development shows these curricula to be more effective than traditional curricula (National Science Foundation, 1995). (In fact, through a broad spectrum of studies, the view of learning and teaching supported by NCTM-related reforms has been scientifically established.)

However, even with these professional recommendation-consistent curricula, teachers with incorrect conceptions of and beliefs about mathematics or about how mathematics is learned can distort the original ideas of the curricula's creators (Battista, 1994; Spillane & Zeuli, 1999). Unfortunately, that is exactly what has been happening. High-quality curricula are being implemented by teachers who have not been properly educated in the curricula's use. In these low fidelity implementations, class discussions often wander aimlessly in free-for-all and show-and-tell styles, ending without students ever coming to closure on ideas (the closure does not have to come at the end of a single period, but it does need to come); the major ideas that students must become fluent with are not properly revisited (perhaps because the teacher doesn't really see how the idea is threaded through a set of activities); and intended instructional goals are not clearly conceptualized and pursued. In fact, it is not unfair to characterize what is happening in these low fidelity implementations of reform curricula as "fuzzy mathematics." The point here is that insufficient teacher preparation and knowledge (concerning mathematics, students' mathematical thinking, and how to conduct high quality inquiry/sense making instruction) is a critical factor in implementations of reform. Without proper training and support, most teachers are unable to properly implement reform practices.

Another factor that contributes to poor implementations of the NCTM Standards is that, because they were meant as guidelines, the Standards were written rather generally to intentionally support variation in instructional practice, within given constraints. An unfortunate consequence of the "general" nature of the Standards is that many school districts claim to be implementing Standards-based mathematics curricula, but their implementations usually have distorted the tenets of the Standards greatly (Hiebert, 1999; NCES, 1996; Spillane & Zeuli, 1999). However, when such implementations fail to live up to expectations, reform mathematics is blamed, even though it is not reform, but a particular implementation of reform, that has failed. Just because a particular implementation fails, one cannot reasonably conclude that the theory and research are wrong, only that there are flawed mechanisms for putting the theory into practice—teacher training and inservice, textbook creation, and teaching.

Teachers with Inadequate Knowledge of Mathematics

As has already been discussed, there is research support and an overall consensus among teacher educators that high quality teaching requires teachers to have sufficient understanding of mathematics (along with student learning and thinking and appropriate pedagogical techniques). Unfortunately, research strongly suggests that teachers' knowledge of mathematics is often shallow, rule-bound, and unconnected (Grouws and Cebulla, in press; also Ma).

> UC Ohio Survey Highlight. 91% of respondents thought that teachers should be periodically tested on their knowledge and teaching of mathematics.

Large Numbers of Unqualified Mathematics Teachers

Despite the documented importance of teacher expertise, "more than 30% of U.S. mathematics teachers were teaching out-of-field in 1991, and only 52% of U.S. mathematics teachers had both a license and a major in their field" (Hammond & Ball, 1997). More than 30% of secondary mathematics teachers do not have even a minor in mathematics. Twenty-seven percent of high school students taking mathematics are taught by out-of-field teachers. Furthermore, “the proportions of students taught by out-of-field teachers are much higher in lower track classes, in high-poverty schools, and high-minority schools" (Hammond & Ball, 1997). In schools with the highest minority enrollments, students have less than a 50% chance of getting a mathematics teacher who holds a license and a degree in the field they teach. There is also a particular problem with the shortage of properly qualified teachers teaching middle school mathematics, with many having only the meager mathematics preparation for elementary education.

The problem of unqualified teachers has been exacerbated whenever states have experienced shortages of mathematics teachers. In such times, states typically permit school districts to use “emergency” regulations to allow unlicensed teachers to teach mathematics. Also, states lower teacher-licensure standards by allowing “alternative” licensing that bypasses established, elaborate university programs. Typically, such alternative programs assume that all teachers need to teach is subject matter knowledge and a bit of hands-on experience in schools, ignoring all the scientific research indicating that teachers also need (a) extensive knowledge of how students learn and how best to teach particular mathematical topics, and (b) knowledge of and competency with instructional techniques that are consistent with professional recommendations and scientific research. In fact, most alternative teacher-licensure programs—with their emphasis on getting teachers into the classroom quickly, and concomitant lack of educational depth—inadvertently promote the type of traditional mathematics instruction that professional recommendations and scientific research have found to be seriously flawed.

Little Systemic Support of Teacher Learning in the Schools

The United States offers far less support for teacher learning than do industrialized countries that rank higher on educational outcome measures. In contrast with countries whose students outperform U.S. students, the United States lacks a genuine professional development system for teachers. "Once in the classroom, U.S. teachers have only 3 to 5 hours a week in which to prepare their lessons, usually in isolation from their colleagues. Most have no time to work with or observe other teachers; they experience occasional hit-and-run workshops that are usually unconnected to their work and immediate problems of practice" (Hammond & Ball, 1997). Indeed, in the U.S., "teachers are assumed to be competent once they have completed their teacher-training programs" (Stigler & Hiebert, 1999, p. 110). In contrast, Japan, which has much higher student achievement in mathematics, makes no such assumption. Participation in continuous, in-depth school-based professional development is considered part of a teacher's job (Stigler & Hiebert, 1999).

Preservice Education in the U.S. Is Weak

Making the problem worse, most teachers in the U.S. also have had a relatively weak program of preservice education. Their undergraduate teacher education programs necessarily make trade-offs between content preparation and pedagogical preparation (generally taught in unconnected courses), and many have weak student teaching experiences. In short, many U.S. teachers enter the profession with inadequate preparation, and few have many genuine opportunities to enhance their knowledge and skills over the course of their careers.

Insufficient amount of Time Spent on Mathematics Teaching

Research also indicates that too little classroom time is being spent on mathematics teaching. In one study, 20% of students at grade 8 had teachers who reported spending less than or equal to 30 minutes per day on mathematics (Grouws & Cebulla, in press). In contrast, many reform-based mathematics curricula require that an hour per day be spent on mathematics teaching.

SECTION 4: MOVING FROM WHERE WE ARE TO WHERE WE WANT TO BE

There are several major changes required to make Ohio mathematics education world-class. These will be summarized below.

There Must Be a Coherent, Shared Vision for Ohio Mathematics Education

One of the major conclusions of TIMSS is that American mathematics education is guided by a disconnected set of restricted views rather than a single overarching coherent vision (Schmidt, McKnight, & Raizen, 1997). Thus, one of the major goals for mathematics education in Ohio should be to develop a coherent vision that guides all aspects mathematics teaching in the state.

As we have seen throughout this document, two fundamental principles form the core of this vision. First, mathematics curricula and instruction should focus on developing in students deep and genuine understanding of mathematical ideas. Depth of understanding should never be sacrificed for breadth of coverage. Second, the only way to instructionally support students' development of mathematical understanding, problem solving, and reasoning, is for curricula and teaching to focus on students' cognitions, not their behaviors. Teaching must guide and support the processes by which students mentally construct personal meaning for mathematical concepts and principles. Design of mathematics curricula and instruction must be grounded in detailed analyses of students' mathematical experiences and the ways that they construct mathematical knowledge. Supporting and guiding students' mathematical cognition can be achieved only by relying on scientific research on mathematics learning.

There Must Be a Comprehensive and Coordinated State Mathematics Education SYSTEM

To implement a coherent and shared vision, Ohio must develop a comprehensive and coordinated state mathematics education system. This system should have at its head a special section of ODE. It should be lead by genuine experts in mathematics education (i.e., accomplished Ph.D.s in mathematics education). It should include active, regular participation by experts in mathematics education research to ensure that policies are scientifically sound. The system should coherently interconnect and guide all components of mathematics education in Ohio, including learning standards, assessment, teacher licensure, university-based teacher education, teacher professional development, and education funding.

However, throughout the development and implementation of the system, we must never lose sight of individual students engaged in the activity of learning. Genuine human learning can never be reduced to a set of standards or test scores. If the mathematics education system becomes a bureaucracy that takes on a life of its own, a life that, once born, loses sight of the human learning activity that it was created to nurture, it will not help improve the situation, but exacerbate it.

Ohio's Mathematics Education System Must Be Firmly Grounded in Scientific Research and Method

All activities and programs carried out by Ohio's mathematics education system must be fully informed by, and consistent with, scientific research in mathematics education. Principles of that research have been outlined in earlier portions of this document. Political whim, education bandwagons, and weakly applicable recommendations of "general" education theory must be avoided. For instance, to insure that state assessment systems and mathematics content standards are consistent with scientific research, development teams must include expert researchers in mathematics learning on a *continuing* basis. (Volunteer work is insufficient and unfair for such continuing involvement; extensive paid consultant arrangements must be made.)

As part of becoming more scientific, all activities in Ohio's mathematics education system must be firmly grounded in the literature in mathematics education. Ohio must utilize the vast resources for improving mathematics education that have been created nationally and internationally. For example, to move toward standards-based mathematics curricula, teachers and mathematics educators in Ohio should not create their own curricula; they should utilize curricula developed by nationally and internationally recognized experts, and funded by the National Science Foundation.

The Mathematics Education System Must Be Centered Around a Set of Scientifically Sound, Professionally Recommended Learning Standards

As discussed in earlier sections, Ohio must have a set of scientifically sound, professionally recommended, cognition-based learning standards in mathematics. The Ohio Model Mathematics Curriculum, which was based on the NCTM Standards written in 1989, was a laudable attempt at creating such standards. However, because NCTM published a revised set of standards in April of 2000, because experience is a good teacher, and because we now have a decade more of relevant research to inform us, it is time to revise Ohio's standards. It is essential that these standards be formulated and documented in a scientifically sound way. Relying solely on professional recommendations, without careful review and documentation of relevant research, will lead to learning standards that are inconsistent with research-grounded instructional practice.

As part of these standards, Ohio must specify what mathematical knowledge is required for graduation from high school and what mathematical knowledge is required for entrance into its universities. Once standards for entering universities are specified, universities in Ohio should be prevented from accepting students who do not meet the standards. (Of course, specific and effective ways should be found to help students who want to go to college meet the entrance requirements. For instance, if students who were enrolled in a college prep program in high school do not meet college entrance requirements, it could be the high school's responsibility to provide the necessary remediation.)

Student Learning Must Be Measured with a Scientifically Sound, Professionally Recommended, Assessment System

Ohio must develop a comprehensive, scientifically sound, professionally recommended, standards-based assessment system. This system must not only inform everyone about how effective the mathematics education system is working, it must provide timely feedback that schools and teachers can use to improve students' mathematics learning. It must include cognition-based assessment of students' learning progress as well as evaluation of students' behavioral competencies. Assessment analyses should be completed quickly enough so that teachers can use the results to help their current-year students.

One way to make Ohio's assessment system more useful is to make it two-tiered. Tier 1 assessments would be given by the State at, say, grades 4, 8 and 12 (the latter possibly being an exit exam that is first given in 10^{th} grade). Tier 2 assessments would be designed/adopted by the State but administered by school districts; given in all grades except 4, 8, and 12; and focus heavily on cognition-based analyses that are critical to guiding instruction. So that individual students' learning progress could be constantly monitored, all tests would be cognition-based and tightly coordinated with the cognition-based learning standards. The assessments, both State and district-administered, would consist not only of multiple-choice items, but open-response and extended-response items. District administered tests would involve teacher committees in most of the grading (especially of open-response and rubric-graded items) so that teachers become intimately aware of, and constantly involved with, the standards-based assessment process. Some Tier 2 assessment items could be embedded in instruction so that they not only serve an assessment purpose but an instructional one.

The key to this assessment system is its coordination with learning standards and instruction. Every time one of the assessments is given, teachers should quickly receive feedback on how individual students are progressing toward the learning standards.

The State should also regularly participate in some nationwide assessments (like the NAEP, and normal standardized tests) to see how Ohio's students are doing in comparison to students in other states.

An Appropriate Teacher Licensure and Teaching Requirement System Must Be Developed

Ohio must develop a licensure/qualification system that ensures that all teachers who teach mathematics know a sufficient amount of (a) mathematics, (b) research on how students learn mathematics, and (c) instructional methods for teaching mathematics, at ALL grades specified by their teaching license.

The United States is one of the few countries in the world that continues to pretend—despite substantial evidence to the contrary—that elementary school teachers are able to teach all subjects equally well. It is time that we identify a cadre of teachers with special interests in mathematics and science who would be well prepared to teach young children both mathematics and science in an integrated, discovery-based environment. (National Research Council 1989, p. 64)

In all but the primary grades (and even there, greatly increased knowledge is needed), increased knowledge requirements demand (as recommended by NCTM) that *mathematics education specialists teach mathematics*.

Although Ohio has made laudable progress in this domain with its new licensure system, the present system has one major gap. The knowledge requirements should apply to all teachers of mathematics, *experienced and new*; no "grandfather" exceptions should be granted, as is currently done. Special inservice programs should be designed to help willing, previously certified teachers move into teaching mathematics full-time. But teachers who do not take advantage of these opportunities should not be permitted to continue teaching mathematics. Moreover, school districts must be required to utilize specialists in mathematics classrooms.

UC Ohio Survey Highlight. 86% of respondents felt that elementary teachers should have specialized training to teach mathematics.

An Appropriate Teacher Inservice Professional Development System Must Be Instituted and Required

Because no undergraduate program can fully prepare teachers to teach mathematics well, Ohio must develop a comprehensive system for the continuous professional development of mathematics teachers that focuses on research-based methods for improving students' mathematics learning. The system should support teachers' regular and collaborative study of mathematics teaching by setting aside significant daily work time for such tasks—time away from "teaching" duties. During this set-aside time, teachers would be responsible for something like the Lesson Study program described by Stigler and Hiebert in the *Teaching Gap*. Involving teachers in the regular study of their own teaching not only is a form of professional development, it can be an essential part of the feedback/adjustment mechanism that continuously improves the functioning of the mathematics education system at the classroom level, where it counts most.

The state should offer leadership, support, and requirements in helping school districts implement continuous professional development programs. There are a number ways that it can do this. First, there can be requirements that, to continue teaching mathematics, mathematics teachers at all levels be required to get a masters degree with a significant amount of coursework appropriate for mathematics education, and that they periodically take "refresher" courses in mathematics education and mathematics throughout their careers. Second, the state could institute requirements for (a) the minimum number of paid hours that schools set aside for teachers to spend on appropriately specified professional development during each school week, and (b) the maximum number of hours that teachers can spend on actual teaching per week.

Operation of the Entire Mathematics Education System Must Be Monitored

Ohio should develop a mechanism to monitor the functioning of the whole mathematics education system. Data on the functioning of the system must be regularly collected and analyzed. An external review of all components of the system by outside experts in mathematics education should be conducted every 5 years. This review should be made public.

Accountability Must Apply to All Components of the System

Accountability should apply not just to schools and teachers, but to state programs (including legislative mandates), universities, school districts, students, and parents. As we have seen, the mathematics education system consists of many components, all of which can profoundly affect student learning. To "blame" teachers or schools by holding only them accountable is intellectually dishonest because it fails to consider that each component can contribute to poor student learning. It is also counterproductive because it diverts attention from factors that may be causing poor learning but are not under the control of schools or teachers. For instance, high-stakes testing that focuses on behavioral rather than cognition-based learning goals drives teachers to teach superficially to the tests, which research shows leads to poor student learning. Thus, blaming teachers and schools for low student achievement diverts attention from the real problem—inadequate assessment and learning standards.

This does not mean that Ohio should not have statewide assessments and standards for learning mathematics. However, the assessments and standards should be consistent with current research on mathematics learning, and teachers should be supported in learning and implementing instructional methods that promote high level mathematics learning in their students.

To improve mathematics learning in Ohio, we must change the whole system; focusing on single components will not work. In fact, since there is sufficient evidence that mathematics learning can be improved by following the tenets provided in this document, the case could be made that all components in the mathematics education system that are inconsistent with these tenets should be blamed for students' poor achievement.

Further Ways the State Can Improve Mathematics Instruction

The state can:

- Require that superintendents and principals take appropriate courses that help them understand research and reform in mathematics education. Such courses can help administrators become part of the solution to the problem, not part of the problem. The state should also institute ways monitor whether administrators are implementing policies that are consistent with these reform principles and research.
- Require that school systems use professionally recommended and scientifically sound mathematics curricula.
- Require that sufficient classroom time be mandated for mathematics instruction each day.
- Require that school districts employ only qualified mathematics teachers for mathematics instruction.
- Require that universities implement scientifically sound, professionally recommended teacher education programs at both the preservice and inservice levels.
- Require that mathematics teaching in universities be consistent with professional curricular and teaching standards.
- Require that increased financial support be given to colleges and universities for mathematics teacher education programs.

The state could also sponsor a systematic program that educates the public about the importance of schooling in general, and mathematics education in particular. Both print materials and TV advertisements should be used to convince students and the public that knowledge of mathematics is essential to personal, community, state, and national well-being.

Business and industry should require that the employees they hire have graduated from high school and have succeeded in appropriate mathematics courses.

> NCREL Highlight: "The Education Commission of the States (ECS) recommends a set of incentives for students, including the following:" (Otto & van der Ploeg, 2001d, p. 2)
>
> - Standards for graduation from high school: Recommendations include requiring students to take a larger core of academic classes and to achieve an aggressive minimum score on end-of-course exams or comprehensive exit exams before graduating.
> - Clear and specific requirements for admission to higher education: Clarifying standards and expectations could persuade students that academic performance in high school matters.
> - Academic requirements for employment or apprenticeship programs: The National Alliance of Business (NAB) campaign Making Academics Count encourages companies of all sizes to ask applicants for high school records as part of the hiring process.

Final Words

Mathematics education is not working as well as it should or could in Ohio. It is disjointed, with no coherent, shared vision. Most teachers are teaching in ways that are inconsistent with recommendations made by professional organizations and scientific research. State learning standards and assessments need revision. And, most importantly, students' mathematics learning is inadequate.

However, there is a ray of hope shining into this gloom. Research and professional organizations have found more effective ways to teach mathematics, and they have clarified the new basic skills for the 21st century. If Ohio can muster the will and wherewithal to implement the curricular, instructional, assessment, and professional development practices recommended by research, great improvements in student learning can result. But change will require a totally different mindset by all stakeholders on all aspects of mathematics education. Current polices and programs must be changed. The vision toward improvement is clear; the question is, do we have the strength and commitment to follow it.

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Appendix A: *Principles and Standards* of the National Council of Teachers of Mathematics (NCTM, 2000)

"The Principles describe particular features of high-quality mathematics education. The Standards describe the mathematical content and processes that students should learn. Together, the Principles and Standards constitute a vision to guide educators as they strive for the continual improvement of mathematics education in classrooms, schools, and educational systems."

Principles

Equity

- Excellence in mathematics education requires equity—high expectations and strong support for all students.

Curriculum

- A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.

Teaching

- Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.

Learning

- Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Assessment

- Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

Technology

- Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.

Standards

"What mathematical content and processes should students know and be able to use as they progress through school? Principles and Standards for School Mathematics presents NCTM's proposal for what should be valued in school mathematics education."

Instructional programs from prekindergarten through grade 12 should enable all students to—

Number and Operations

- understand numbers, ways of representing numbers, relationships among numbers, and

number systems;
- understand meanings of operations and how they relate to one another;
- compute fluently and make reasonable estimate

Algebra
- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

Geometry
- analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement
- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability
- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

Problem Solving
- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Reasoning and Proof
- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Communication

- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

Connections

- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

Representation

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.