

**Quantitative Methods for *All* Students  
and  
Why Mathematics Can't Do it Alone**

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# **Quantitative literacy affects:**

- **The decisions students make in their personal and professional lives**
- **Students' understanding of the political and social issues that are part of citizenry: The way they vote, and the policies they support.**
- **The majors students choose and the careers that are open to them.**

# **Chair of a pharmacology department in a medical school:**

“The idea of using a computer for something other than email or downloading documents to print is one that only a small fraction of our class is comfortable with. I would say the same thing about things numerical.....The number of incidents due to miscalculated drug doses becomes more understandable.”

**Quantitative literacy matters!**

# What is Quantitative Literacy?

- **The ability to identify, understand, and use elementary mathematics in everyday contexts.**
- **Fluency in adapting this ability to new contexts.**
- ***A habit of mind*; not a particular piece of knowledge or a particular skill.**
- **Looking for quantitative patterns everywhere**

# Deaths in Two Tragedies

<i>Event I</i>	Deaths
Men	1364
Women	126

<i>Event II</i>	Deaths
Men	89
Women	284

- More than 10 times as many men died
- Original population  $\frac{2}{3}$  men
- Sinking of Titanic, April 14, 1912
- Almost 4 times as many women died
- Original population half women
- Tsunami, village in Indonesia, December 26, 2004

**Mathematics: Ratios and Approximation**

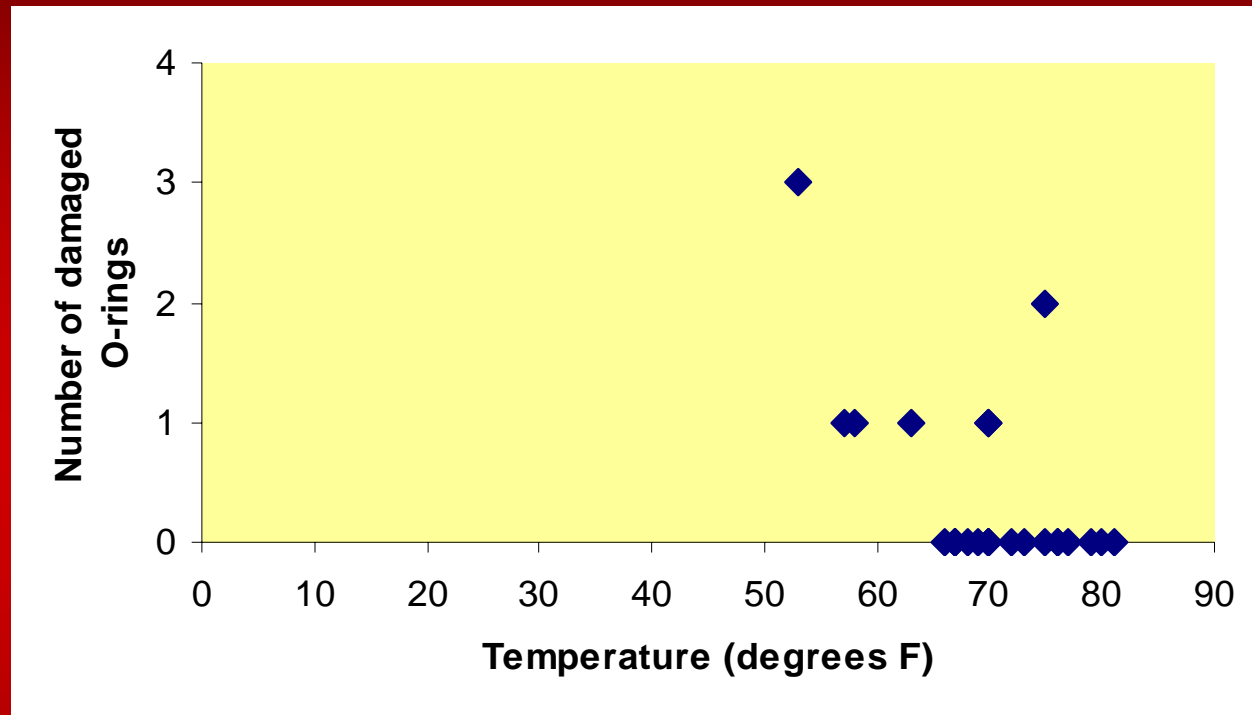
# On January 28, 1986, Space Shuttle Challenger Exploded

- **What was cause of disaster?**
- **In early 1987, Richard Feynman showed the following data to a congressional hearing**
- **This data was available to NASA in January 1986.**

Temp (F)	O-rings damaged	Temp (F)	O-rings damaged
53	3	70	1
57	1	70	1
58	1	72	0
63	1	73	0
66	0	75	0
67	0	75	2
67	0	76	0
67	0	76	0
68	0	77	0
69	0	79	0
70	0	80	0
70	0	81	0
70	0		

**What could this data have told NASA?**

# A Graph Shows the Effect of Temperature on O-Rings



- O-rings more likely to break at low temperatures
- Temperature on January 28, 1986, was 31°F

Mathematics: Graphing, Exploratory Data Analysis

# Is Quantitative Literacy Different from Mathematical Literacy?

## Mathematical Literacy: Requires

- Wide knowledge of mathematics
- Ability to abstract away from context

## Quantitative Literacy: Requires

- Fluency in elementary mathematics
- Ability to think in context

# **Is It Possible to be Mathematically Literate Without Being Quantitatively Literate?**

- **A certain level of mathematics is needed for quantitative literacy.**
- **However, it is possible to have learned the mathematics without being able to apply it in context.**

**What does this mean for teaching and curriculum?**

# Strike Over Gas Prices in UK

In fall 2000, caller to *The Talk of the Nation*<sup>1</sup> pointed out that a tax of £8 out of £10 spent on gas is a 400% tax, not the 80% tax that the British Government was claiming.<sup>2</sup>

- Was the caller mathematically literate?
- Was the caller quantitatively literate?

1. PBS, September 18, 2000.

2. BBC web page: BBC News/World Fuel Crisis/UK Fuel Tax: The Facts, September 21, 2000.

## **UK Gas Prices:**

**£8 out of £10 spent on gas was tax**

**A group of US students who heard this story knew what percent 8 was of 10, and what percent 8 was of 2, but not where the caller got 400% and 80%.**

- Were these students mathematically literate?**
- Were these students quantitatively literate?**

# What Mathematics is Needed for Quantitative Literacy?

People can reasonably differ. A possible list:

- Arithmetic, including percentages, graphs.
- Estimation.
- Elementary probability and statistics.
- Basic geometry of measurement (volume, area, perimeter).
- Elementary growth patterns: linear (constant quantity per unit time) and exponential (constant percentage per unit time).

This has significant implications for teaching in schools

# **Conceptual Underpinnings**

## ***Every Student Needs***

**How to “read” mathematical information:**

- **Numbers, graphs, symbols, units**
- **Translating between different forms promotes understanding**

# **Conceptual Underpinnings: Numbers**

- **Relative sizes of numbers**
- **Approximate mental calculations**
- **Ability to see trends in data**
- **Units**

# Numbers: What is the Trend?

- 2, 4, 6, 8, 10
- 2.13, 4.06, 5.92, 7.81, 9.89
- 2, 4, 8, 16, 32
- 1.96, 3.97, 8.21, 15.77, 32.11

**In which of these is the pattern hard to see? Why?**

Estimation and Mental Arithmetic; Linear and Exponential trends

# Is There Racial Profiling in LA?

LAPD motor vehicle stops resulting in search  
(01/01/04-06/30/04)

## Race of driver

	White	African-American	
Search	5849	9882	15731
No search	106104	49439	155543

# Approximating the Data

	Race of driver		
	White	African-American	
Search	6,000	10,000	16,000
No search	110,000	50,000	160,000

# Numbers in Thousands

- About 5% of white drivers were searched when stopped
- About 20% of African-American drivers were searched when stopped

	Race of driver		
	White	African-American	
Search	6	10	16
No search	110	50	160

# Conceptual Underpinnings: Graphs

- **Understanding the story told by a graph**
- **Seeing trends on graphs**
- **Knowing what the intercepts and asymptotes represent**

The cost  $C$  and revenue  $R$  functions for a certain company are shown in Figure 2.34.

If the company is currently producing 750 units, which of the following is true?

- (a) The company is making a profit and should increase production.
- (b) The company is not making a profit and should increase production.
- (c) The company is making a profit and should decrease production.
- (d) The company is not making a profit and should decrease production.

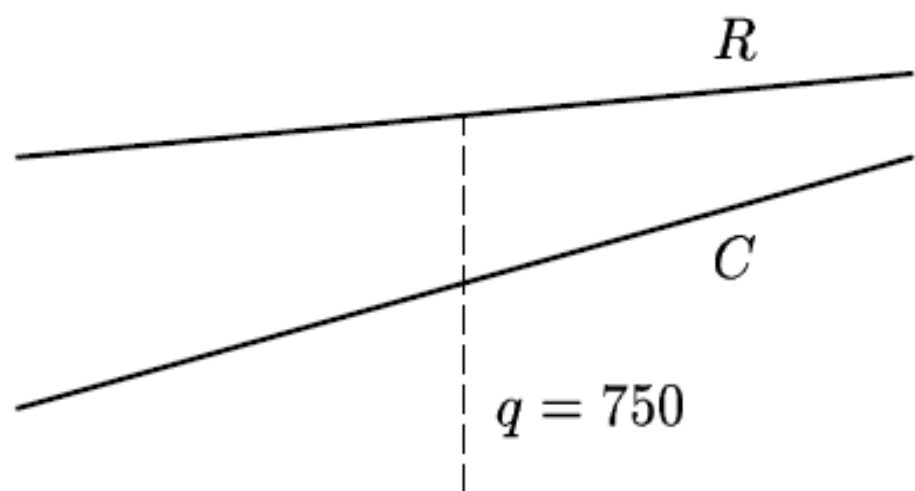


Figure 2.34

The graphs show the temperature of yams put in an oven at time  $x = 0$ . Which yam

(a) Is in the warmest oven?

(b) Started at the lowest temperature?

(c) Heated up fastest?

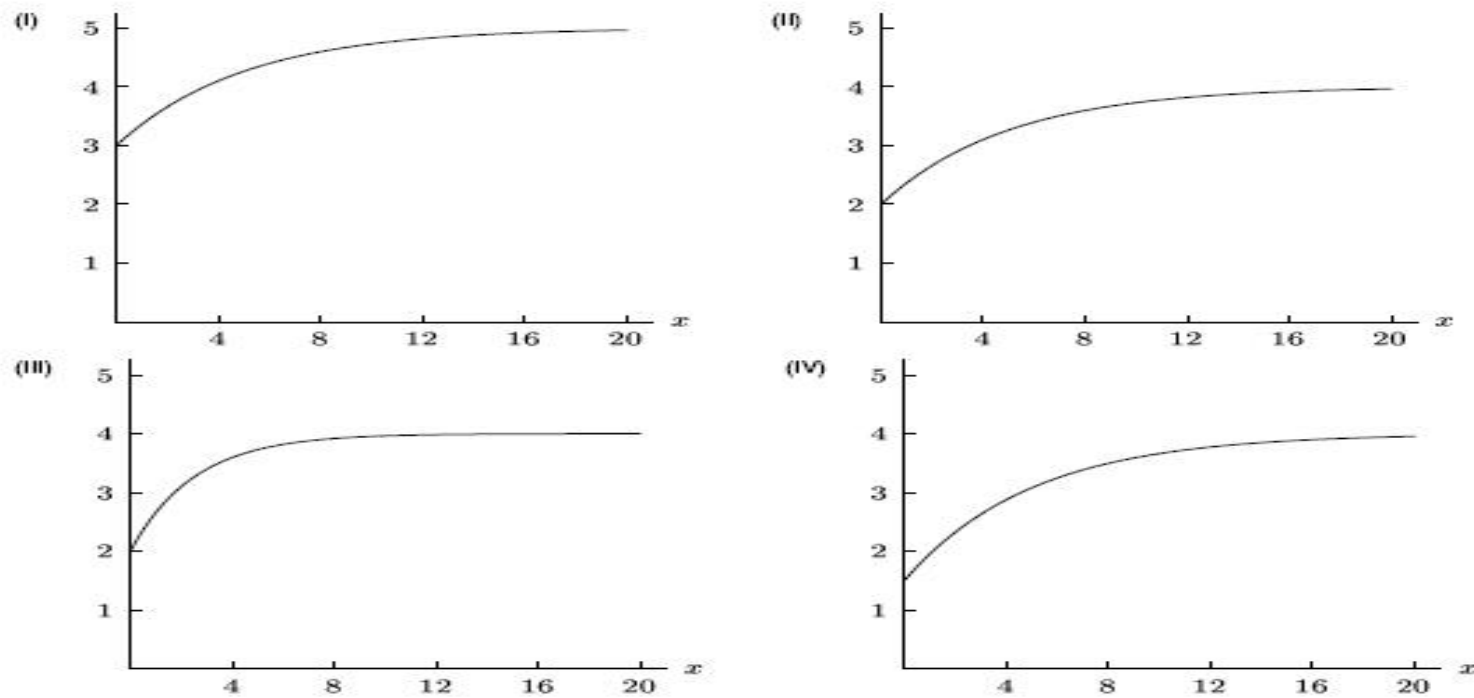


Figure 1.7

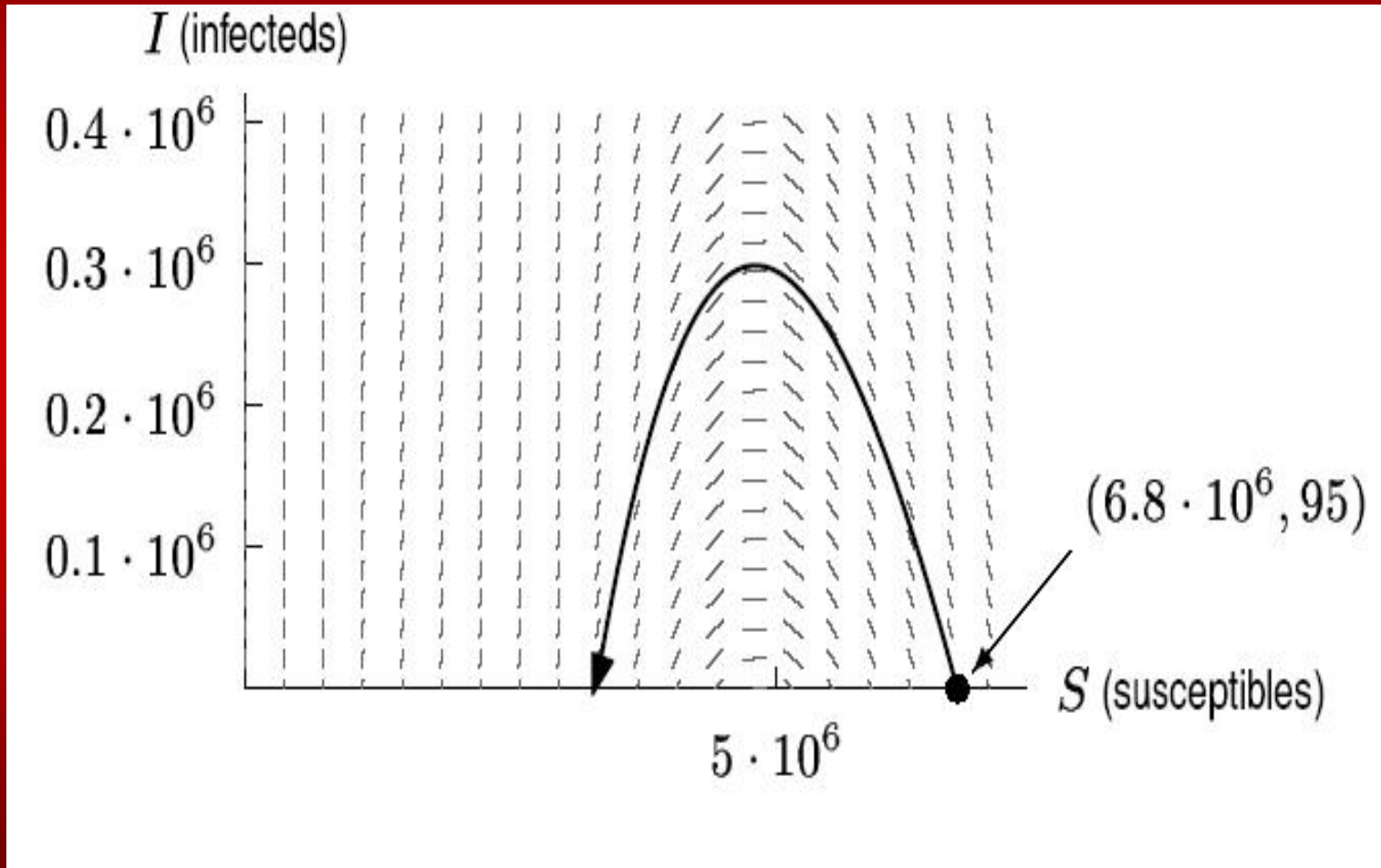
# **Preventing the Spread of SARS**

**There is an outbreak of SARS in a nearby city. As you are working in the mayor's office, you are asked which is the most effective policy for preventing the spread to the disease to your city:**

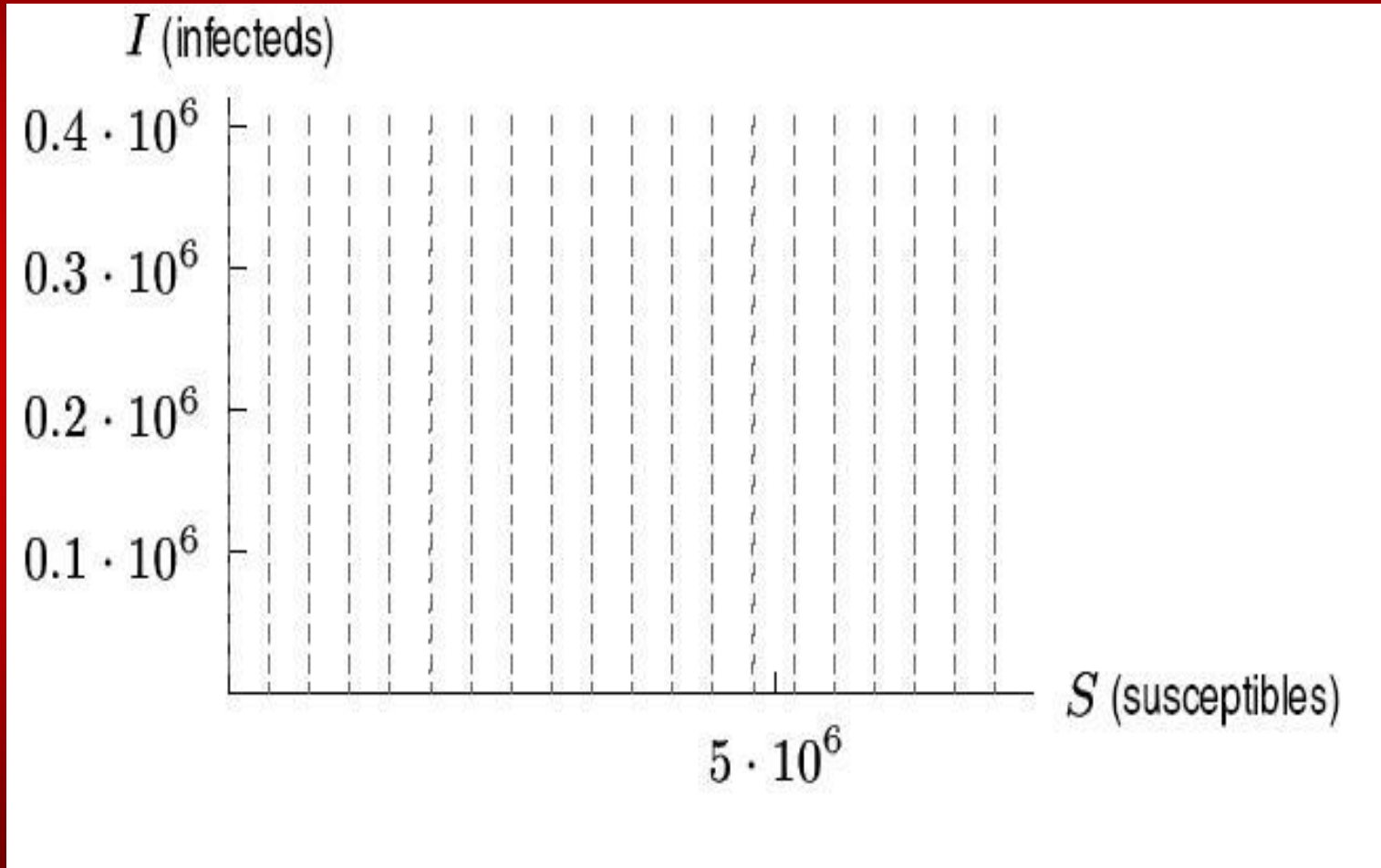
- I. Close off the city from contact with the infected region. Shut down roads, airports, trains, busses, and other forms of direct contact.**
- II. Install a quarantine policy. Isolate anyone who has been in contact with a SARS patient or anyone who shows symptoms of SARS.**

**(Analyze using 2003 World Health Organization data from Hong Kong)**

# No quarantine



# With quarantine



# Big Ideas: Rate of Change

- Function: Shows how two quantities are related
- Calculus: Shows how changes in one quantity are related to changes in another quantity
- Rates of Change: Enable projections

# **Rates of Change: Linear and Exponential Growth**

- **The Earth's temperature increased 0.36 degrees Fahrenheit in 30 years:**
- **Prices rose by 2.5% last year:**

# **Rates of Change: Linear and Exponential Growth**

- **The Earth's temperature increased 0.36 degrees Fahrenheit in 30 year:  
Linear**
- **Prices rose by 2.5% last year:  
Exponential**

# Rates of Change: Calculus

If  $f(t)$  is the population of the US in millions at time  $t$  in years, what is the meaning of the statements  $f(2006) = 300$  and  $f'(2006) = 2.87$ ?

How do these statements relate the BBC's observation<sup>1</sup> that a new person is added to the US population every 11 seconds?

- **Some calculus students could answer; some could not.**

1. BBC web page: BBC News/Americas/US population reaches 300 million. October 17, 2006

# Big Ideas: Variability

- Probability: Measures the likelihood of an event
- Statistical Inference: What judgments can we make based on the data we observe? How variable do we expect data to be?

# Should We Have Widespread HIV Testing?

## In the US

- HIV tests are very accurate (over 99%)
- Yet the probability of a positive HIV tests being correct is only about 15%
- Why? Is testing a bad idea?

Probability

# Some Further Questions about LAPD Data

(Numbers in Thousands)

- About 5% of white drivers were searched when stopped
- About 20% of African-American drivers were searched when stopped

	Race of driver		
	White	African-American	
Search	6	10	16
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**Mathematics: Estimation and Percentage**

## Further Questions about LAPD data

- Could the differences in percentages have happened by accident if there has been no racial profiling?
- Is the difference between 5% and 20% large enough to provide evidence that the LAPD uses racial profiling?
- The data was for January 1- June 30, 2004. Might a different period have shown substantially different percentages?

# **The Human Cost of the War in Iraq: Lancet Mortality Study 2002-2006**

- **Estimate of 654,965 excess deaths since 2003**
- **Cluster sample to estimate death rate  
(deaths/1000/year)**
- **95% confidence interval for violent deaths:  
(426,369, 793,663)**

**Mathematics: Rates, Proportion, Sampling, Inference**

# How Should We Teach Quantitative Literacy?

- **Students see mathematics as useful only when they see it used in other fields**
- **Many applications should involve making a decision (not a calculation).**
- **Faculty should model quantitative literacy themselves.**
- **Coordination between faculty teaching quantitative literacy is important.**

# **How Do We Improve Quantitative Literacy?**

- **How do we get students to use quantitative methods in diverse contexts?**
- **It takes a “conspiracy”**
- **How does one foster such a conspiracy?**
- **Your ideas?**

**Thank You!**